Discrete Fourier Transforms and Z-Transforms

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April 17, 2024

1 Results & Discussion

1.1 The Discrete Fourier Transform (DFT)

Given a signal, $x[n]$, it's N-point DFT is given by

$$X_k = \sum_{n=0}^{N-1} x[n] W_N^{kn}, \tag{1}$$

where $W_N = e^{-j2\pi/N}$. The discrete Fourier transform is the sampled version of the discrete time Fourier transform (DTFT), which is a continuous function. More specifically, the N-point DFT contains N samples from the continuous DTFT.

For example, consider the signal $x[n] = (-1)^n$ for $0 \leq n \leq N-1$. By evaluating the sum shown in equation 1 as a truncated geometric series, the N-point DFT of $x[n]$ can be found. All truncated geometric series are evaluated as

$$\sum_{k=0}^{n-1} ar^k = \begin{cases} an & r = 1 \\ a\left(\frac{1-r^n}{1-r}\right) & r \neq 1 \end{cases}, \tag{2}$$

where r is the common ratio between adjacent terms. For the N-point DFT of $x[n]$, the common ratio is $-W_N^k$, which takes a value of 1 for $k = \frac{N}{2}$. Therefore, the N-point DFT of $x[n]$ is

$$X[k] = \begin{cases} N & k = \frac{N}{2} \\ \left(\frac{1-(-W_N^k)^N}{1-(-W_N^k)}\right) & k \neq \frac{N}{2} \end{cases}. \tag{3}$$

The N-point DFT of $x[n]$, where $N = 8$ is seen in figure 1. It only has a non-zero value for $k = \frac{N}{2} = 4$. This is the case for all even-number-point DFTs. Therefore only odd-number-point DFTs should be used. For example, the 9-point DFT of $x[n]$, where $N = 8$ is seen in figure **??**. While equation 3 cannot be used

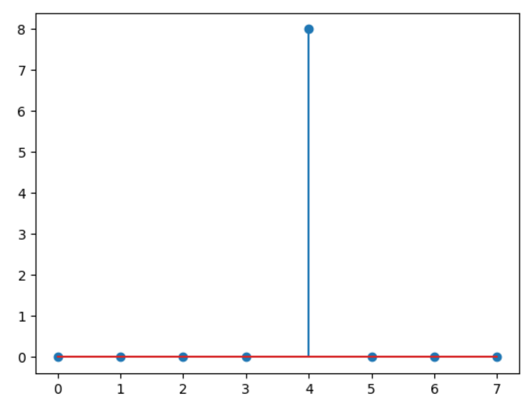


Figure 1: The N-point DFT of $x[n]$, where $N = 8$

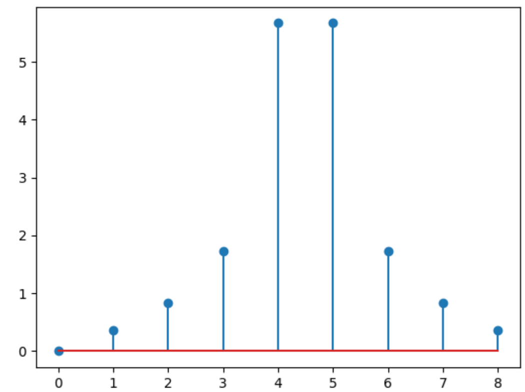
because there are a different number of samples for the DFT and the input signal, the overall DFT is more useful than the 8-point DFT.

1.2 The Z-Transform

Given a discrete signal, $x[n]$, its z-transform is given by

$$X(z) = \sum_{n} x[n] z^{-n} \tag{4}$$

where z is a complex variable.



1.3 The Inverse Z-Transform

2 Conclusions