

Homework 2 - Aidan Sharpe

1

Consider two **disjoint** events V and W and suppose we know the probabilities $P(V)$ and $P(W)$. What is a simple expression for $P(V \cap W^c)$, using only the information given? You may obtain the answer by drawing a Venn diagram or listing intermediate equations.

$$P(W^c) = 1 - P(W)$$

If V and W are disjoint, then V and W^c must be joint. For joint probabilities:

$$P(A \cap B) = P(A)P(B)$$

Combining the above:

$$P(V \cap W^c) = P(V)[1 - P(W)]$$

2

Consider a portion of an electric circuit that includes three independent components, each of which can either be open or closed. In order for current to flow from point A to point B, at least one of these components must be closed. Each component has a probability of 0.85 of functioning properly and closing when activated.

a)

What is the probability that the current will flow from point A to point B?

Let R_1 be the event that relay 1 works, R_2 be the event that relay 2 works, R_3 be the event that relay 3 works, and C be the event that current flows.

$$P(C) = P(R_1 \cup R_2 \cup R_3)$$

$$P(C) = P(R_1) + P(R_2) + P(R_3) - P(R_1 \cap R_2) - P(R_1 \cap R_3) - P(R_2 \cap R_3) + P(R_1 \cap R_2 \cap R_3)$$

$$P(R_1) = P(R_2) = P(R_3) = 0.85$$

$$P(R_1 \cap R_2) = P(R_1 \cap R_3) = P(R_2 \cap R_3) = 0.722$$

$$P(R_1 \cap R_2 \cap R_3) = 0.614$$

Plugging in yields:

$$P(C) = 0.9966$$

b)

What is the probability that relay 2 functioned, given that the current flowed from point A to point B?

$$P(R_2|C) = \frac{P(C|R_2)P(R_2)}{P(C)}$$

Since current will always flow if relay 2 works:

$$P(C|R_2) = 1$$

$$\therefore P(R_2|C) = \frac{P(R_2)}{P(C)} = 0.853$$

3

Selena uses one of her four cars to drive to work everyday. She drives one and only one car to work each day.

$$P(C_1) = 0.4$$

$$P(C_2) = 0.3$$

$$P(C_3) = 0.15$$

$$P(C_4) = 0.15$$

Let X be the event that Selena will arrive to work on time. Depending on the car used, the probability of X varies.

$$P(X|C_1) = 0.85$$

$$P(X|C_2) = 0.9$$

$$P(X|C_3) = 0.8$$

$$P(X|C_4) = 0.8$$

a)

Find $P(C_2 \cup C_3)$:

Since C_2 and C_3 are disjoint events:

$$P(C_2 \cup C_3) = P(C_2) + P(C_3) = 0.45$$

b)

$$P(C_1|X) = \frac{P(X|C_1)P(C_1)}{P(X)}$$

$$P(X) = P((X|C_1) \cup (X|C_2) \cup (X|C_3) \cup (X|C_4)) = 0.85$$

$$P(C_1|X) = \frac{0.85 \times 0.4}{0.85} = 0.4$$

4

Suppose you are given four copies of the novel *Burns: The Expertise of Achievement*, which has become a very popular read amongst America's urban youth. The four copies are known to contain two defectives. Your job is to test the books one at a time until the two defective books are found. Once you locate the two defective copies, you will stop testing. Let X denote the number of the test on which the second defective copy of the book is found. Find the probability distribution for X .

$$P(X = 2) = \frac{1}{6}$$

$$P(X = 3) = \frac{1}{3}$$

$$P(X = 4) = \frac{1}{2}$$

5

The probability distribution of X , the number of imperfections per 10 meters of a synthetic fabric in continuous rolls of uniform width, is given by: $|x|f(x)$ |0|0.41 |1|0.37 |2|0.16 |3|0.05 |4|0.01

a)

What is the probability that there are 3 imperfections?

$$P(X = 3) = 0.05$$

b)

What is the probability that there is at least one but no more than three imperfections?

$$P(X = 1 \cup X = 2 \cup X = 3) = 0.37 + 0.16 + 0.05 = 0.58$$

6

Software to detect fraud in consumer phone cards tracks the number of metropolitan areas where calls originate each day. It is found that 1% of the legitimate users originate calls from two or more metropolitan areas in a single day. However, 30% of fraudulent users originate calls from two or more metropolitan areas in a single day. The proportion of fraudulent users is 0.01%. If the same user originates calls from two or more metropolitan areas in a single day, what is the probability that the user is fraudulent?

F is the event that a user is fraudulent, F^c is the event that a user is not fraudulent. M is the event that a user originates calls from two or more metropolitan areas in a single day.

$$P(F) = 0.0001$$

$$P(F^c) = 0.9999$$

$$P(M|F) = 0.3$$

$$P(M|F^c) = 0.01$$

$$P(M) = 0.3 \times 0.0001 + 0.01 \times 0.9999 = 0.01003$$

$$P(F|M) = \frac{P(M|F)P(F)}{P(M)} = \frac{0.3 \times 0.0001}{0.01003} = 0.00299$$

7

An assembly consists of three mechanical components. Suppose that the probabilities that the first, second, and third components meet specifications are 0.95, 0.98, and 0.99. Assume that the components are independent. Determine the probability mass function of the number of components in the assembly that meet specifications.

Let C_1 be the event that the first component meets specification, C_2 be the event that the second component meets specification, C_3 be the event that the third component meets specification, and X be the number of components that meet specification:

$$P(X = 0) = P(C_1 \cap C_2 \cap C_3) = 0.95 \times 0.98 \times 0.99 = 0.9217$$

$$P(X = 1) = P((C_1 \cap C_2 \cap C_3^c) \cup (C_1 \cap C_3 \cap C_2^c) \cup (C_2 \cap C_3 \cap C_1^c))$$

$$P(X = 1) = 0.0766$$

$$P(X = 2) = P((C_1 \cap C_2^c \cap C_3^c) \cup (C_2 \cap C_1^c \cap C_3^c) \cup (C_3 \cap C_2^c \cap C_1^c))$$

$$P(X = 2) = 0.00167$$

$$P(X = 3) = P(C_1^c \cap C_2^c \cap C_3^c) = 0.05 \times 0.02 \times 0.01 = 0.00001$$

$$p_X(x) = \begin{cases} 0.921689 & x = 0 \\ 0.07663 & x = 1 \\ 0.00167 & x = 2 \\ 0.00001 & x = 3 \end{cases}$$