Homework 8 - Aidan Sharpe

1

A transmission line is terminated with a matched 50Ω load. The transmitter puts out 100W of power, and the transmission line is 100ft long. What for what value of α will the power loss be 10W over the length of the line?

$$\alpha = \frac{\ln\left(\frac{P_{\text{in}}}{P_{\text{out}}}\right)}{l} = \frac{\ln\left(\frac{100}{90}\right)}{30.48[\text{m}]} = 0.00345671 \left[\frac{\text{np}}{\text{m}}\right]$$

2

Evaluate the phase velocity and attenuation constant for a distorionless line, and compare it to a lossless line.

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$
$$\gamma^2 = (R + j\omega L)(G + j\omega C)$$
$$\gamma^2 = \omega^2 LC(j + \frac{R}{\omega L})(j + \frac{G}{\omega C})$$

For a distorionless line:

$$\frac{R}{L} = \frac{G}{C}$$
$$(j + \frac{R}{\omega L}) = (j + \frac{G}{\omega C})$$

So γ^2 becomes:

$$\gamma^2 = \omega^2 LC(j + \frac{R}{\omega L})^2$$

Solving for γ

$$\gamma = \omega\sqrt{LC}(j + \frac{R}{\omega L})$$

Separating the real and imaginary components:

$$\alpha = R\sqrt{\frac{C}{L}}$$
$$\beta = j\omega\sqrt{LC}$$

For a lossless line, there is no attenuation (by definition). Therefore:

$$\alpha = 0$$

To actually be able to build this lossless line, $R = 0$ and $G = 0$.

$$\gamma^2 = (R + j\omega L)(G + j\omega C)$$

Since R and G are both 0:

$$\gamma^2 = (j\omega L)(j\omega C)$$

$$\gamma^2 = -\omega^2 LC$$

$$\gamma = \sqrt{-\omega^2 LC} = j\omega\sqrt{LC}$$

Separating out real and imaginary:

$$\alpha = 0$$

$$\beta = j\omega\sqrt{LC}$$

For lossless and distortionless lines, the attenuation constant differs, but the phase constant does not. Since the phase velocity only depends on ω and β, v_p is the same for both lossless and distortionless lines.

$$v_p = \frac{\omega}{\beta} = \frac{-j}{\sqrt{LC}}$$

3

$$\hat{Z}_L = 75 + j150\Omega$$

$$f = 2[\text{MHz}]$$

$$\omega = 2\pi f = 4\pi \times 10^6$$

$$r = 150 \left[\frac{\Omega}{\text{km}}\right]$$

$$l = 1.4 \left[\frac{\text{mH}}{\text{km}}\right]$$

$$c = 88 \left[\frac{\text{nF}}{\text{km}}\right]$$

$$g = 0.8 \left[\frac{\mu\text{s}}{\text{km}}\right]$$

$$\hat{V}_G = 100e^{j0^\circ}$$

$$z = 100[\text{m}]$$

$$R = 15[\Omega]$$

$$L = 140[\mu\text{H}]$$

$$C = 8.8[\text{nF}]$$

$$G = 80[n\Omega]$$

a)

Find $\hat{Z}_0$:

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{15 + j(4\pi \times 10^6)(140 \times 10^{-6})}{80 \times 10^{-9} + j(4\pi \times 10^6)(8.8 \times 10^{-9})}}$$

$$Z_0 = 126.1324 - j0.5377$$

$$\hat{\gamma} = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\hat{\gamma} = 0.0595 + j13.9482$$

b)

Find the input imedance

$$Z_{\text{in}} = Z_0 \frac{Z_L + Z_0 \tanh(\gamma z)}{Z_0 + Z_L \tanh(\gamma z)}$$

$$Z_{\text{in}} = -126.1324 + j0.5377$$

c)

Find the average power delivered

$$\alpha = \frac{\ln\left(\frac{P_{\text{in}}}{P_{\text{out}}}\right)}{z}$$

$$\alpha = 0.0595$$

$$P_{\text{in}} = \frac{V_G^2}{Z_{\text{in}}} = 79.2802 + j0.3373$$

$$P_{\text{out}} = P_{\text{in}} e^{-\alpha z} = 0.2073 + j0.0009$$

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$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

a)

$$Z_L = 3Z_0$$

$$\Gamma = 0.5$$

b)

$$Z_L = (2 - j2)Z_0$$

$$\Gamma = 0.5385 - j0.3077$$

c)

$$Z_L = -j2Z_0$$
$$\Gamma = 0.6 - j0.8$$

d)

$$Z_L = 0$$
$$\Gamma = -1$$

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a)

$$\Gamma = 0.06 + j0.24$$

b)

$$\text{VWSR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 1.657$$

c)

$$Z_{\text{in}} = Z_0 \frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)} 30.50 - j1.09$$

d)

$$Y_{\text{in}} = \frac{1}{Z_i n} = 0.03274 + j0.0012$$

e)

$$0.106\lambda$$

f)

$$z = -0.106\lambda$$

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$$L = \frac{3\lambda}{8}$$
$$Z_{\text{in}} = -j2.5$$
$$Z_L = \frac{-j2.5}{100} = -j0.025$$

At $\frac{3\lambda}{8}$:

$$Z_L = j95$$