Homework 3 - Aidan Sharpe

1

An particle with charge $-e$ has velocity $\vec{v} = -v\hat{y}$. An electric acts in the $\hat{x}$ direction. What direction must a $\vec{B}$ field act for the net force on the particle to be 0?

$$\vec{F}_{net} = \vec{F}_E + \vec{F}_B = 0$$
$$\therefore \vec{F}_B = -\vec{F}_E$$
$$\vec{F}_E = q\vec{E} = (-)(\hat{x}) = -\hat{x}$$
$$\vec{F}_B = q\vec{v} \times \vec{B}$$
$$(-)\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & (-) & 0 \\ B_x & B_y & B_z \end{vmatrix} \stackrel{!}{=} \hat{x}$$
$$-[(-B_z)\hat{x} - (0)\hat{y} + (0 - (-)B_x)\hat{z})] \stackrel{!}{=} \hat{x}$$
$$\therefore \hat{a}_B = \hat{z}$$

2

Consider a plane wave in free space with electromagnetic field intensity:

$$\hat{E}e^{j\omega t} = 30\pi e^{j(10^8 t + \beta z)}\hat{x}$$
$$\hat{H}e^{j\omega t} = H_m e^{j(10^8 t + \beta z)}\hat{y}$$

Find the direction of propagation and the values for H_m, β, and the wavelength.

Since βz is added to ωt, propagation is in the $-\hat{z}$ direction.

$$\frac{E_x}{H_y} = \mu_0 c$$
$$E_x = 30\pi$$
$$H_y = H_m$$
$$\therefore H_m = \frac{30\pi}{\mu_0 c}$$
$$\boxed{H_m = 0.25}$$

$$\beta = \omega\sqrt{\mu\varepsilon}$$

For free space:

$$\beta = \omega\sqrt{\mu_0\varepsilon_0}$$

$$\omega = 10^8$$

$$\boxed{\beta = 0.334}$$

$$\lambda = \frac{c}{f}$$

$$f = \frac{\omega}{2\pi} = \frac{5}{\pi} \times 10^7$$

$$\lambda = \frac{3 \times 10^8}{\frac{5}{\pi} \times 10^7} = \frac{30\pi}{5}$$

$$\boxed{\lambda = 6\pi[\text{m}]}$$

3

A uniform electric field has intensity:

$$\vec{E} = 15\cos\left(\pi \times 10^8 t + \frac{\pi}{3}z\right)\hat{y}$$

The $\vec{E}$ field is polarized in the $\hat{y}$ direction.

The wave will propagate in the $-\hat{z}$ direction.

$$f = \frac{\pi \times 10^8}{2\pi} = 5 \times 10^7[\text{s}^{-1}]$$

$$\lambda = \frac{3 \times 10^8}{5 \times 10^7} = 6[\text{m}]$$

$$H_x = \frac{15}{\mu_0 c} = 0.0398$$

$$\vec{H} = 0.0398\cos\left(\pi \times 10^8 t + \frac{\pi}{3}z\right)\hat{x}$$

4

a)

The properties of a uniform basic plane wave in free space are:

Polarization, amplitude, angular frequency, and the direction of propagation. All other properties, such as wavelength, and the spatial frequency, β, can be derrived.

b)

A uniform plane wave in free space is propagating in the $\hat{z}$ direction. If the wavelength is $\lambda = 3$[cm].

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{3 \times 10^{-2}} = 10^{10}[\mathrm{s}^{-1}]$$
$$\beta = 2\pi f \sqrt{\mu_0 \varepsilon_0} = 209.613$$

The amplitude of the x-polarized $\vec{E}$-field is:

$$\hat{E}_m = 200 e^{j\frac{\pi}{4}}$$

Real-time $\vec{E}$-field:

$$\vec{E}(z,t) = 200 \cos\left(2\pi \times 10^{10} t - 209.613 z + \frac{\pi}{4}\right)\hat{x}$$

Phasor $\vec{H}$-field:

$$\hat{H} = 0.53 e^{j(\frac{\pi}{4} - 209.613z)}\hat{y}$$

Real-time $\vec{H}$-field:

$$\vec{H}(z,t) = 0.53 \cos\left(2\pi \times 10^{10} t - 209.613 z + \frac{\pi}{4}\right)\hat{y}$$

5

A 25[cm] by 25[cm] circuit in the y-z plane grows in the $\hat{y}$ direction by 10[m/s]. The circuit contains a 5-ohm resitor. Find the current through the circuit if it is placed in a uniform $\vec{B}$ field of $-0.5\hat{x}$[T].

By Ohms law:

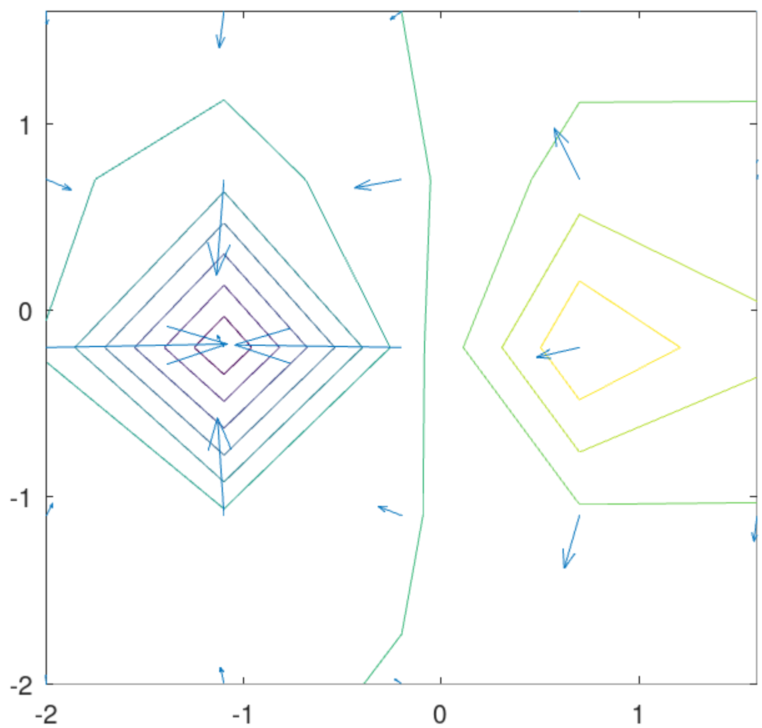
$$V = IR$$
$$\mathcal{E} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{s}$$
$$\mathcal{E} = -\frac{dBA(t)}{dt}$$
$$BA(t) = -0.5 \times 0.25(0.25 + 10t) = -1.25t - 0.03125$$
$$\mathcal{E} = -\frac{dBA(t)}{dt} = 1.25[\mathrm{V}]$$
$$I = \frac{V}{R} = 250[\mathrm{mA}]$$

Since the magnetic flux inside the loop is increasing, and the magnetic field induced by the current must counteract the magnetic field inducing the current, the current must flow counter-clockwise around the loop.

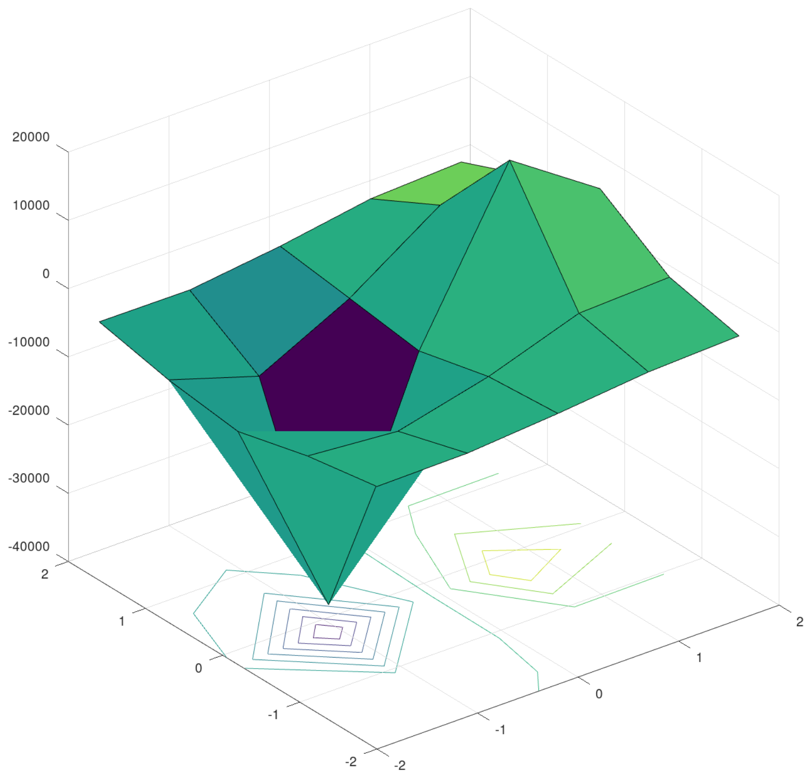
6

a)

```
clear all;
k = 9e9;
q1 = 1.0e-6;
q2 = 1.0e-6;
ax = 1.0;
ay = 0;
bx = -1.0;
by = 0;

[X, Y] = meshgrid(-2:0.9:2,-2:0.9:2);

V = 1./sqrt((X - ax).^2 + (Y-ay).^2 ) * k * q1 + 1./sqrt((X-bx).^2 + (Y-by).^2) * k * q2;

surfc(X, Y, V);

[Ex, Ey] = gradient(-V, 0.2, 0.2);
figure
contour(X, Y, V);

hold on;
quiver(X, Y, Ex, Ey);
```


1
0
-1
-2
-2
-1
0
1

b)



c)

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% Removed the negative sign on q2
q1 = 1.0e-6;
q2 = 1.0e-6;
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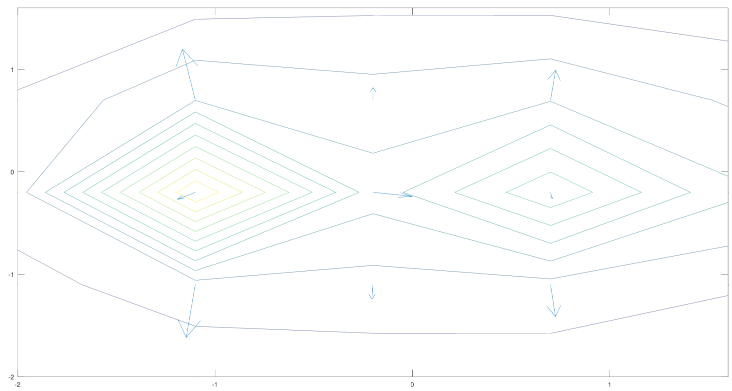

1
0
-1
-2
-2
-1
0
1