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Abstract

1 Introduction

2 Results & Discussion

2.1 Uniform Convergence

Consider the discrete time signal, $x[n] = a^n u[n]$, where $u[n]$ is the unit-step and $a = 0.9$. Let $X(e^{j\omega})$ be the discrete time Fourier transform (DTFT) of $x[n]$. For the DTFT of a signal to exist, it must be absolutely summable over all $n \in \mathbb{Z}$. If the signal is absolutely summable, then it must also be bounded, meaning $|x[n]| < \infty, \forall n \in \mathbb{Z}$. In other words, $x[n]$ must have a finite maximum value.

In this case, $x[n]$ is bounded because a^n grows with smaller values of n, and $u[n]$ is zero when $n < 0$. Therefore, the maximum value of $x[n] = a^n u[n]$ is $x[0] = a$.

Additionally, $x[n]$ takes the form of a geometric series, so its sum is given by

$$\sum_{n=m}^{\infty} s^n = \frac{s^m}{1-s}, |s| < 1. \tag{1}$$

In this case, $m = 0$, because the signal has no value for $n < 0$. Since $a = 0.9$, the sum evaluates to is $\frac{1}{1-0.9} = 10$. Considering that $x[n]$ is always a positive real number, each term is its own absolute value, so the sum and the absolute sum are equivalent. By taking the absolute sum of the first 200 terms of $x[n]$, it becomes clear that it approaches 10 in the limit, seen in figure 1.

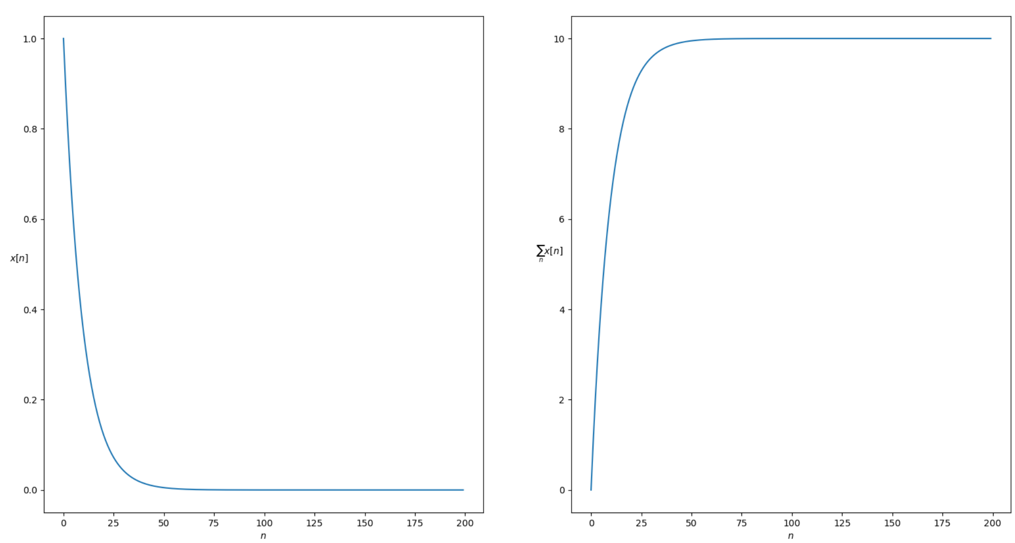

Figure 1: The signal value of $x[n] = a^n u[n]$ and its absolute sum

3 Conclusions