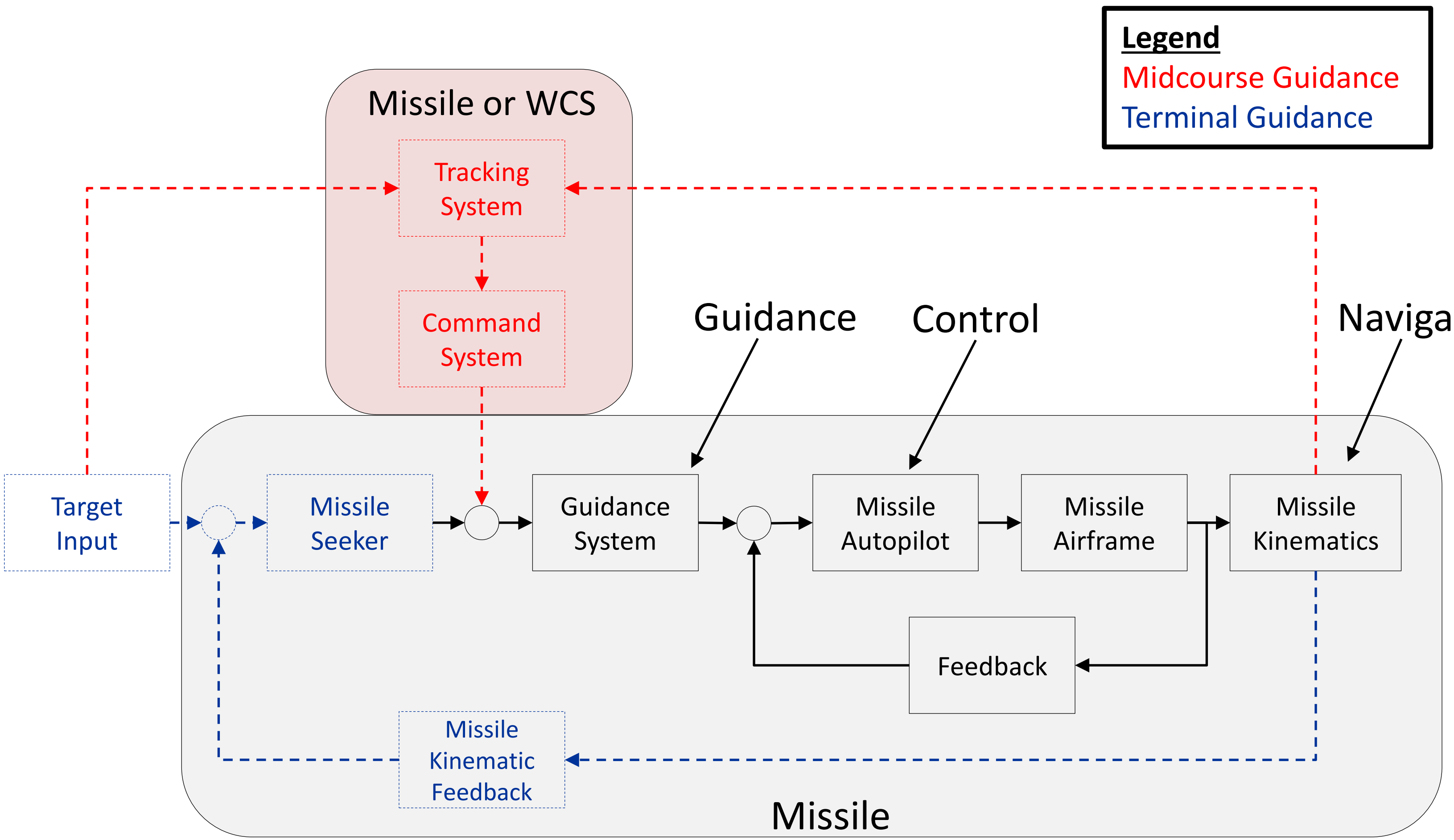
Guidance

Gregg Bock

Closed Loop Guidance

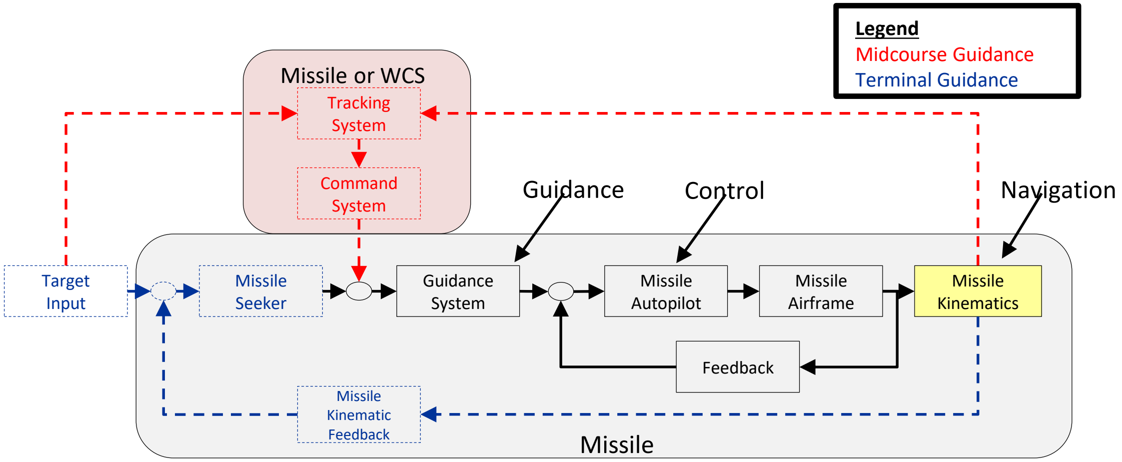
- The closed loop guidance system is responsible for maintaining the interceptor's trajectory along a given path
- The closed loop guidance system is comprised of three systems work together to deliver the interceptor to its intended location
  - Guidance system
  - Navigation system
  - Control system
- Each of the three systems has a unique set of tasks but the codependency of the systems is so strong that the three systems are often referred to as a single entity – Guidance, Navigation, and Control (GNC)

Closed Loop Guidance System



Navigation System

- Determines missile kinematic state information
  - Missile position and its derivatives
  - Angular orientation and its derivatives
- Provides missile kinematics to the guidance system and autopilot



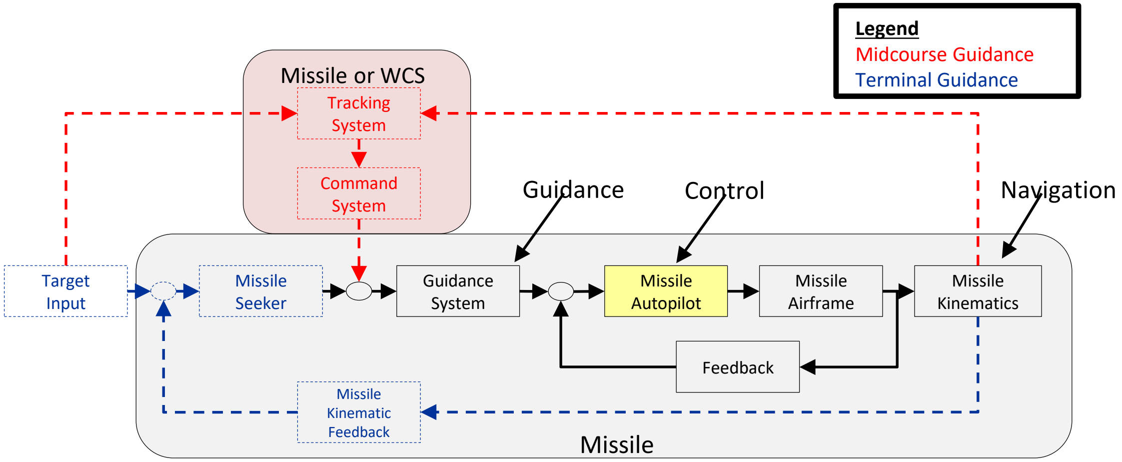
Importance to the WCS Designer

Navigation System

- From a weapon system design perspective, guidance and control are more interesting than the navigation system of the interceptor
  - Choice of trajectory and airframe responsiveness can be influenced (to some degree) by the weapon system designer
- Navigation system errors are considered inputs to the weapon system design
  - The navigation system errors must be considered in the seeker search volume
  - The navigation system may influence the means by which data is provided to the interceptor
    - Certain reference frames may have less error than others

Control System

- Often referred to as the **autopilot**
- Responds to orders from the guidance system to steer the missile onto the intended trajectory
- Feedback loop with the navigation system is used to maintain missile stability and achieve desired flight path



Responsibilities

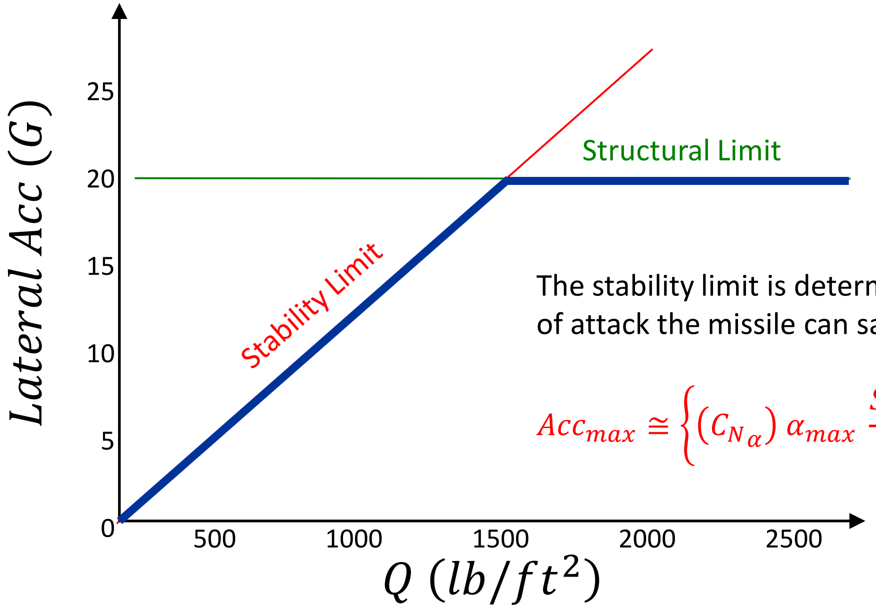
Control System

- Maintains proper flight attitude
  - Roll
    - Rolling airframe missile
    - Roll stabilized missile
  - Flight path angle (pitch and yaw planes)
- Maintain airframe stability
  - Requires the autopilot to achieve a desired command over time
    - This is achieved through the missile autopilot
    - The autopilot uses a time delay based upon environmental and kinematic conditions and to ensure missile airframe stability
  - The maximum acceleration must not exceed the structural limits of the missile
    - Exceeding the structural limit of the missile could tear the missile apart

Maximum Acceleration

Control System

- The maximum lateral acceleration achieved by the interceptor is determined by either stability (maximum angle of attack) or the structural limitation of the missile
- The control system is responsible for ensuring that the interceptor never exceeds the maximum lateral acceleration for the current flight conditions
  - This is often referred to as G-limiting



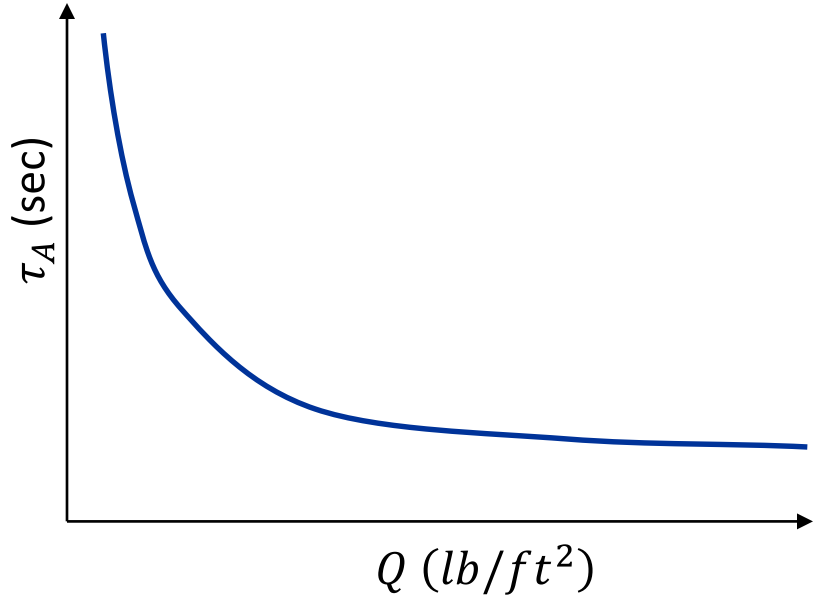
The stability limit is determined by the maximum angle of attack the missile can safely achieve

$$Acc_{max} \cong \left\{ \left(C_{N_\alpha}\right) \alpha_{max} \frac{S_{Ref}}{W} \right\} Q$$

Missile Time Constant, τ_A

Control System

- Indicates the time delay between the observance of the trajectory deviation and the response in terms of the control system output
- It is a function of Mach and altitude
- Can be approximated as a function of dynamic pressure
- Provides a rule of thumb for the time which must be allotted for terminal homing
- Autopilot type (1st order, etc.) typically dictates the number of τ_A required for terminal homing
  - Typical number is between 5 and 10



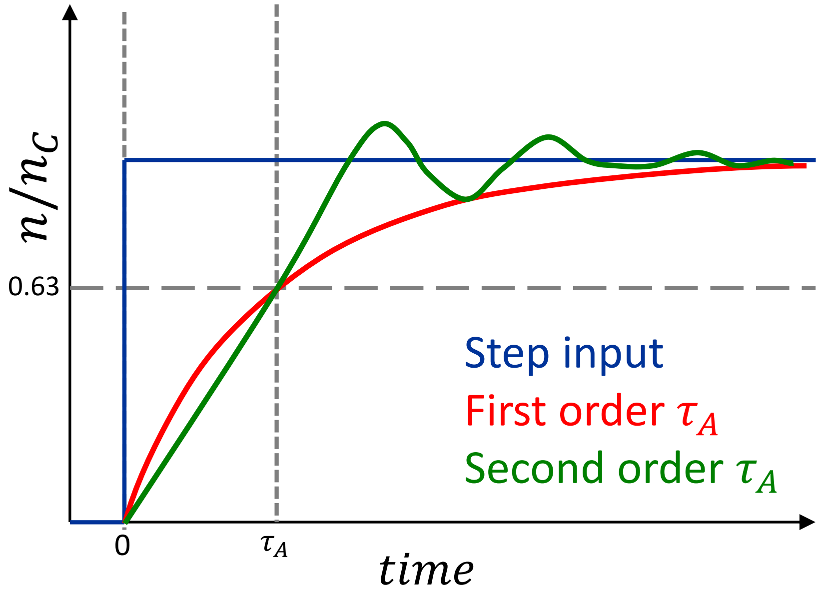
Missile Time Constant, τ_A

Control System

- The missile time constant can be first order or higher (typically not more than fifth order)
- Weapon system designers approximate τ_A as a first order response to an input

$$\frac{n}{n_C} = \frac{1}{s\,(\tau_A)+1}$$

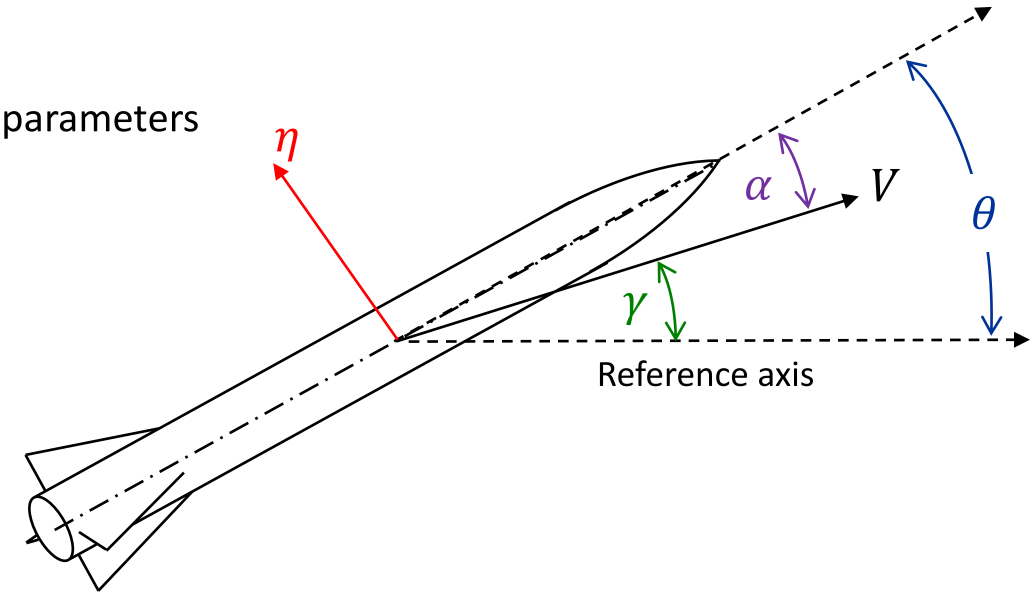
- The time constant is often determined by measuring the response to a step input command at various flight conditions
- A low autopilot response time (τ_A) is required when engaging maneuvering threats, but also increases missile sensitivity to noise in the guidance loop



Control Variables

Control System

- There are four typical control types used to steer the missile
  - Normal acceleration (η)
  - Attitude (θ)
  - Angle of attack (α)
  - Flight path angle (γ)
- Sensors used to measure control parameters
  - Linear accelerometers
  - Angular accelerometers
  - Attitude gyros
  - Rate gyros



Methods of Control

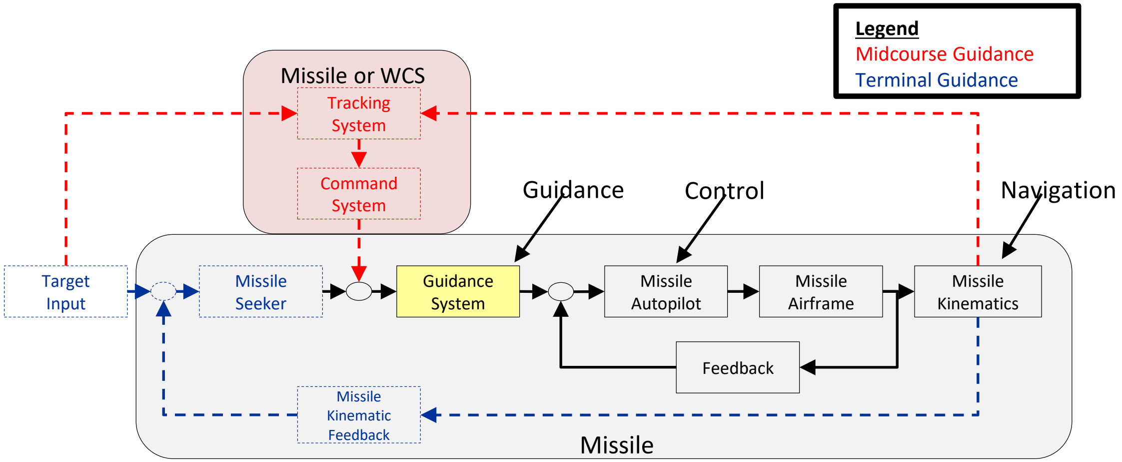
Control System

Control	Control Variable	Inertial reference Required	Measurement Method
Acceleration	η	☒	Accelerometer
Turning Rate	$\dot{\theta}$	☒	Rate gyro
Angle of attack	α	☑	α
Flight path angle	γ	☑	θ and α
Attitude *	θ	☒	θ or rate gyro

* Attitude control is critical for roll stabilization

Guidance System

- Keep the missile along the intended course (trajectory)
- The trajectory to be flown is determined via the guidance law



Responsibilities

Guidance System

- Determine the guidance mode
  - Terminal
  - Midcourse
  - Initial
- Consider the relative missile-target geometry in order to compute the desired flight path
  - Missile receives information from via a communications link, or through its own sensors
  - Data is used to make decisions regarding a future trajectory
    - Intercept point prediction (to where is the missile flying?)
    - Trajectory restrictions/limitations/requirements
- Compute the corrections required to fly the intended trajectory
- Direct flight path corrections be made in the form of acceleration commands
  - Commands are issued to the autopilot

Evolution of Missile Guidance

Guidance System

- Guidance laws are used to determine the desired missile path from its current location to its final location

- The earliest rocket/missile guidance law utilized pre-programmed flight paths
  - Missile flies a preprogrammed trajectory
  - Relies upon accelerometers and gyros to ensure the proper trajectory is flown
    - Required very little missile navigation errors to ensure intercept
  - No course corrections possible due to lack of relative target-missile position updates

- Soon after, missiles were equipped with measuring devices capable of receiving an electro-magnetic signal (radar, interferometer, camera, etc.)
  - Provides real-time estimates of missile and/or target kinematic states measured during flight
  - State estimates are used to update missile guidance commands
    - Remove missile navigation system errors
    - Correct for target movement

Missile Guidance Laws

Classic

Non-Homing

Position / orientation of interceptor relative to natural landmarks (stars, etc.) are used to compute guidance commands. Note that the intercept point is a point that can always be described relative to natural landmarks such as celestial bodies, terrain, etc.

Intuitive

Simple guidance algorithms designed to drive the missile to intercept based upon common sense and/or maritime experience, etc.

Modern

Optimal Control

Optimal control guidance laws consider optimizing a cost (final interceptor speed or miss distance) while often times considering additional constraints to the optimization problem.

Differential Games

Differential game theory considers an intelligent target which is trying to avoid the interceptor. This results in a two-side optimal control problem

Other Branches

Other branches of modern guidance consider multiple hypothesis target models, fuzzy logic in guidance law selection or guidance gain criteria, or applying principles of other scientific research to the guidance problem

Predictive Guidance

Predictive guidance laws consider the target's trajectory to be known. A target's trajectory is considered in the intercept geometry to generate guidance commands

Missile Guidance Laws

Classic
1940s-1960s

- Classic guidance laws were developed based upon intuition, practical experience, and common sense
- The only goal of a classic guidance law is to hit the target

Non-Homing

Preset (preprogrammed)

Celestial
- Guidance based upon location of the celestial bodies to determine missile kinematic states

Terrain
- Landmarks in the terrain are used to update missile kinematic states

Homing

Intuitive Guidance Laws

Line-of-Sight (LOS)
- Beam Rider
- Constant LOS

Pursuit
- Pure Pursuit
- Lead Pursuit

Maritime Guidance Laws

Constant Bearing

Proportional Navigation (PN)
- PN
- Pure

Non-Homing Guidance

Missile Guidance Laws

- Earliest forms of guidance used reference points to update the ***missile*** states in the guidance equation
  - The interceptor would determine its position relative to known reference points
  - Location of reference points in relation to missile position and orientation is used to determine a flight path
- Celestial guidance
  - Missile uses the celestial bodies (stars, planets, etc.) as known reference points
  - Similar to how sailors navigated across oceans before GPS
- Terrestrial guidance
  - Terrain maps are used as reference points rather than celestial bodies

Non-Homing Guidance Provides Capability Against Stationary Targets

Homing Guidance

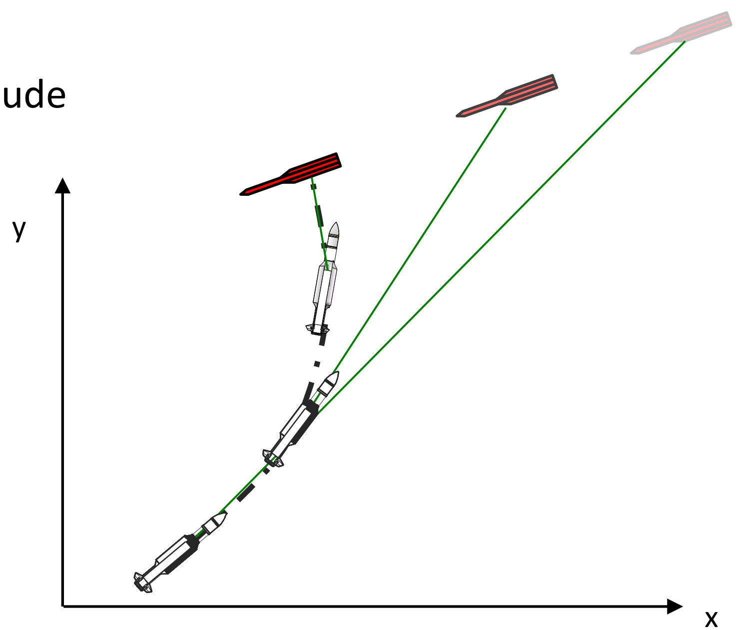
Missile Guidance Laws

- Measuring devices would to be used to determine the location of the intended target rather than using natural landmarks to determine the missile's position and orientation
  - Tracking the intended target allows the guidance design engineer to update the target location in the guidance loop
  - Guidance laws can "correct" for target movement over time
- Restrictions on computer processing speed, electronic power (wattage) required simple guidance laws
  - Simple to implement
- Lack of maturity in optimization theory did not allow for more complex guidance laws
  - Simple design
- Assumptions
  - Constant missile speed
  - Constant target velocity
  - Small angle approximations

Intuitive Guidance Laws

Homing Guidance Laws

- Early guidance schemes were designed to force an intercept by flying the missile along the line-of-sight of the radar tracking the target and the target (λ)
- Little, or nothing, regarding the targets course and speed were considered when generating guidance commands
  - The missile only reacted to the current line-of-sight (λ)
  - Poor performance against crossing targets was inevitable as there is no "lead angle" consideration in the guidance law
- Examples of intuitive guidance laws include
  - Beam rider guidance
  - Pursuit guidance



Beam Rider

Synopsis

- Missile flies by maintaining a trajectory within a beam that is pointed at the target (RF or LASER signal)
- Missile travels within the "beam", but with an oscillatory motion as it tries to center itself within the beam
- Drawbacks
  - As intercept range increases, missile accuracy decreases due to beam dispersion
  - Poor performance when engaging crossing targets
  - Significant WCS resource requirements (continuous illumination of the target)

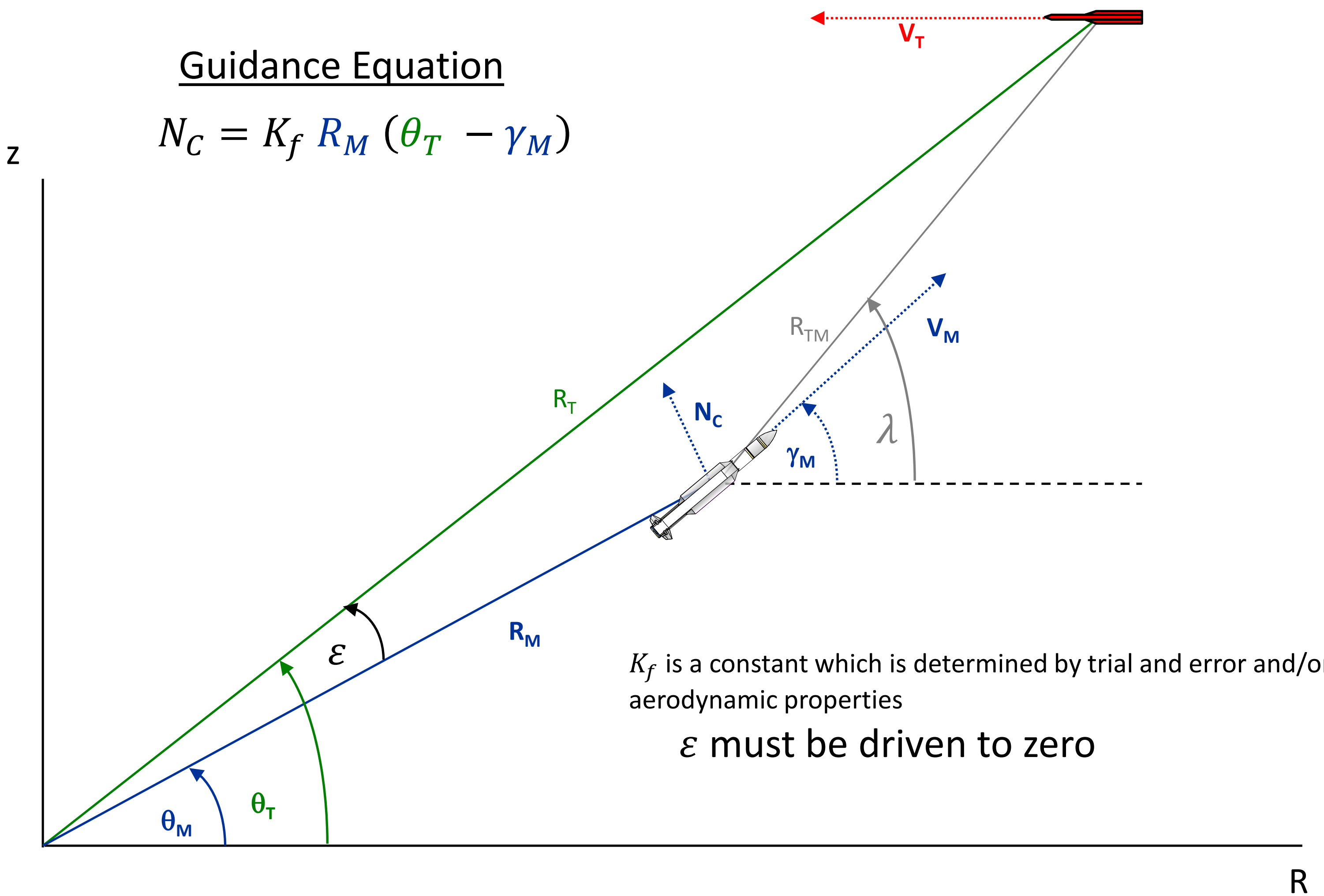
Beam Rider Guidance Results in a Tail Chase When Engaging Crossing Targets

Guidance Law

Beam Rider

<u>Guidance Equation</u>

$$N_C = K_f \, R_M \, (\theta_T - \gamma_M)$$



K_f is a constant which is determined by trial and error and/or missile aerodynamic properties

ε must be driven to zero

Pursuit Guidance

Synopsis

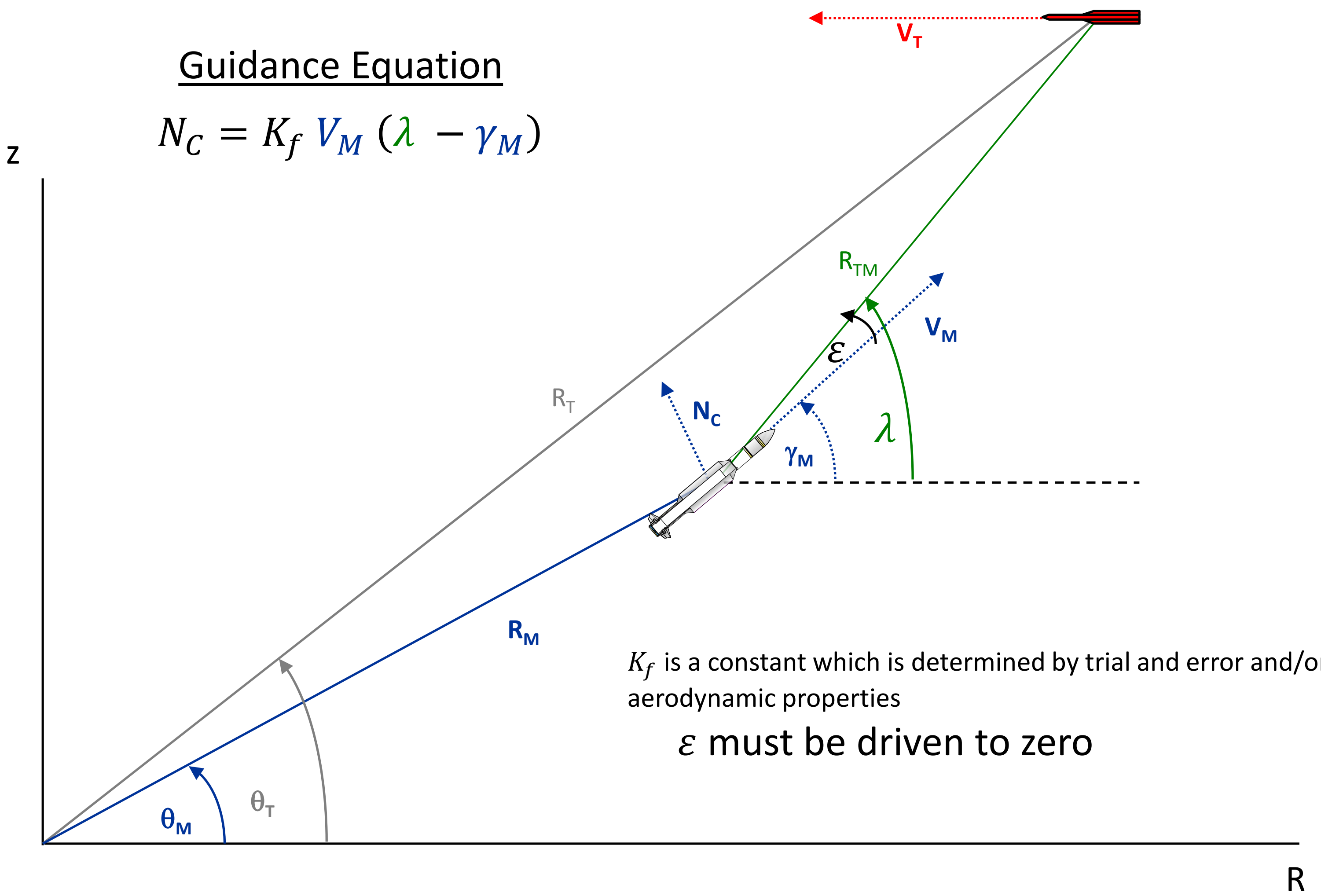
- Also referred to as “Hound and Hare” guidance or “Pure Pursuit” Guidance
- Missile is guided in a manner by which the missile velocity vector is pointed at the current target position
  - If the missile is pointed at the target, eventually an intercept must occur
- Drawbacks
  - Like its name implies, missile will “pursue” targets, resulting in tail chases, in all but the most favorable geometries
- Variations
  - Pure Pursuit Guidance
    - Attitude Pursuit Guidance
    - Velocity Pursuit Guidance
  - Deviant/Lead Pursuit Guidance

Guidance Law

Pursuit Guidance

Guidance Equation

$$N_C = K_f \, V_M \, (\lambda - \gamma_M)$$



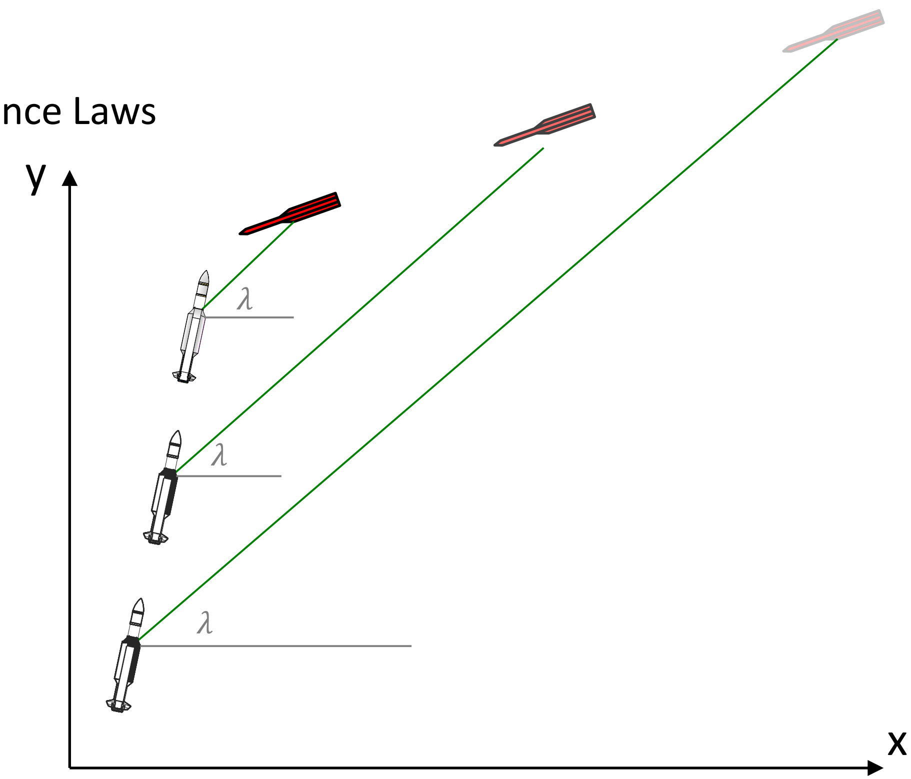
K_f is a constant which is determined by trial and error and/or missile aerodynamic properties

ε must be driven to zero

Maritime Influenced Laws

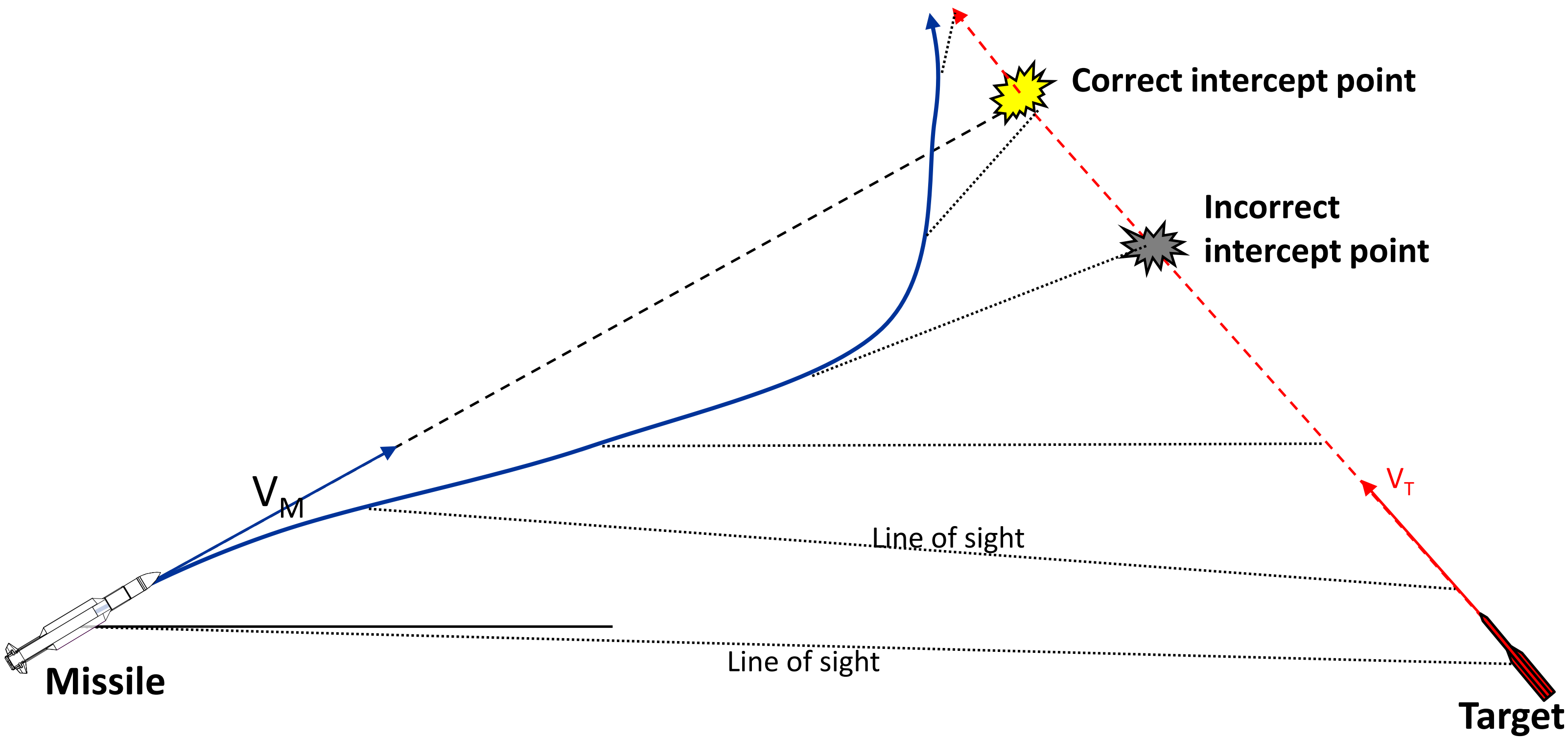
Intuitive Guidance Laws

- Captains of naval vessels have long understood that a collision course is guaranteed if the relative line of sight between two ships (λ) is constant
  - The same holds true for missiles, automobiles, go-karts, etc.
- This crude but effect concept is utilized with far greater success than the guidance laws that did not consider target motion
- Examples of Maritime Influenced Guidance Laws
  - Constant Bearing
  - Proportional navigation



Avoiding a Collision

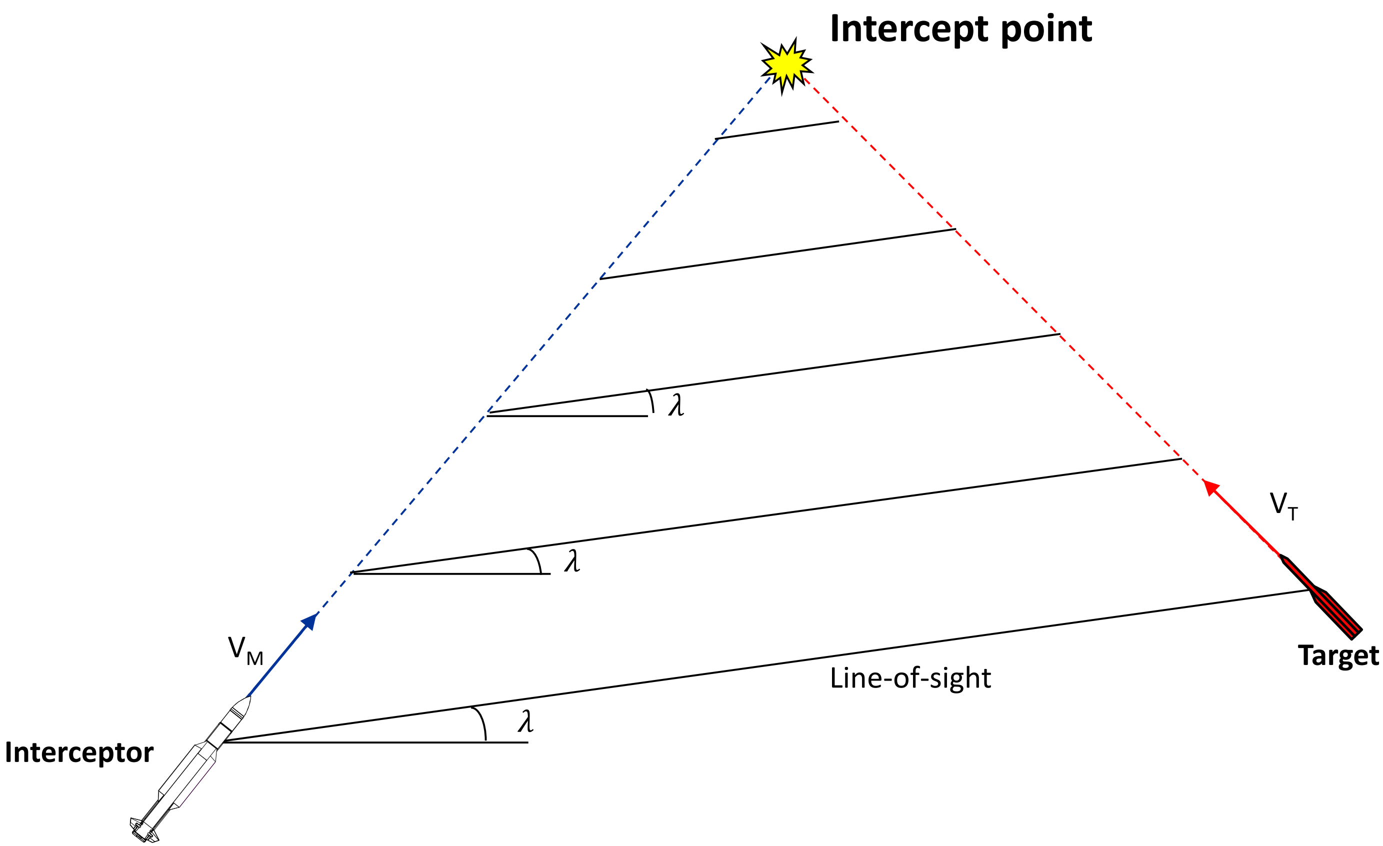
Maritime Influenced Guidance Laws



Note how the line-of sight changes through out the flight

Achieving a Collision

Maritime Influenced Guidance Laws



Note how the line-of sight changes remains constant through out the flight

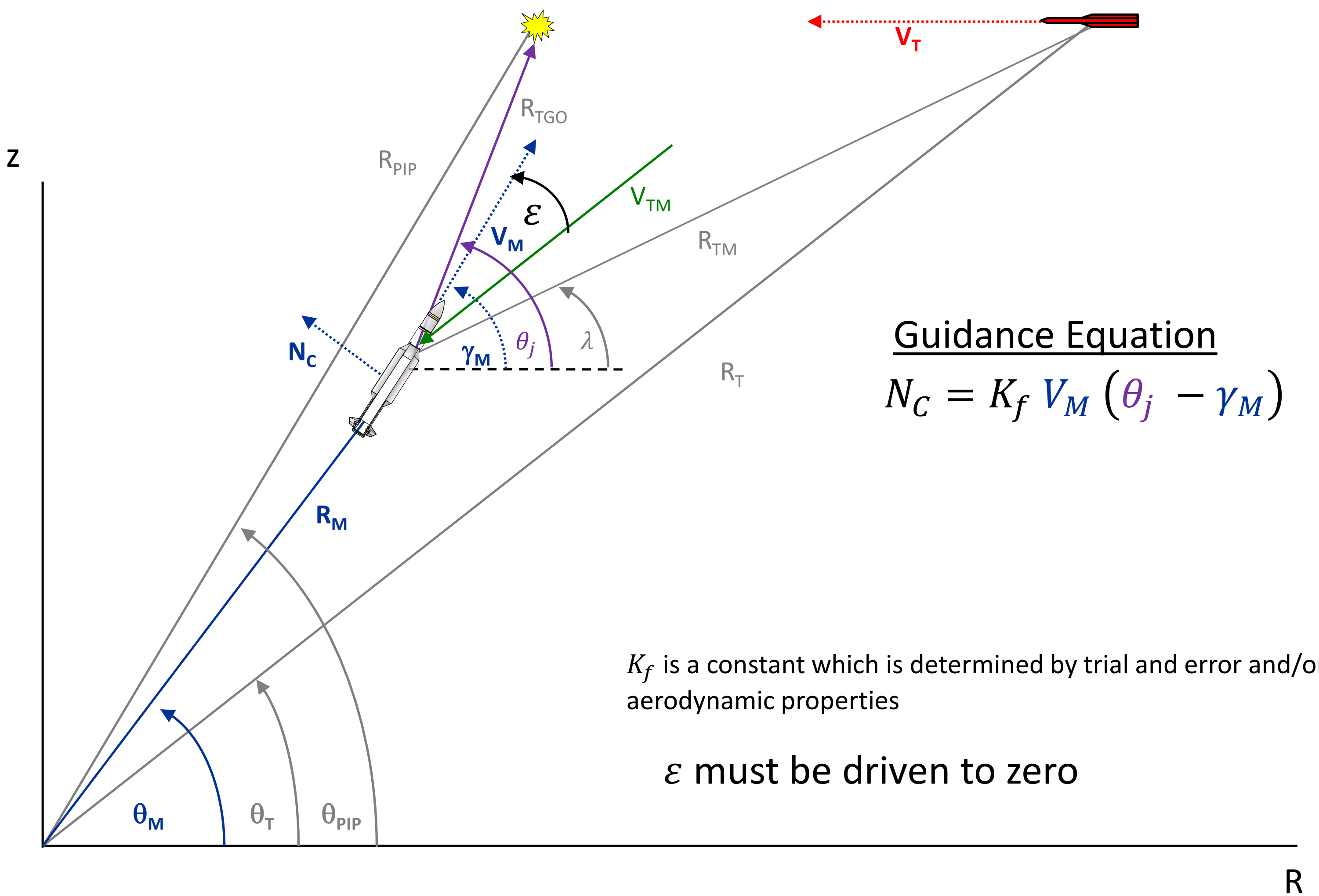
Constant Bearing

Synopsis

- Also known as "parallel navigation" or "collision course navigation"
- The missile is aimed at a point ahead of the target, calculated to be where both the missile and target will arrive at the same instant.
  - Requires missile velocity and target velocity to be constant
- Enough information regarding the target and the missile must be available to the weapon control system to predict future positions as a function of time
- Drawback:
  - If the target or missile velocity changes, a new collision course must be computed and the missile flight path altered accordingly
  - Missile is often flown "in plane" with line-of-sight to target to reduce course corrections

Guidance Law

Constant Bearing



Guidance Equation

$$N_C = K_f\, V_M\, (\theta_j \; - \gamma_M)$$

K_f is a constant which is determined by trial and error and/or missile aerodynamic properties

ε must be driven to zero

Proportional Navigation (PN)

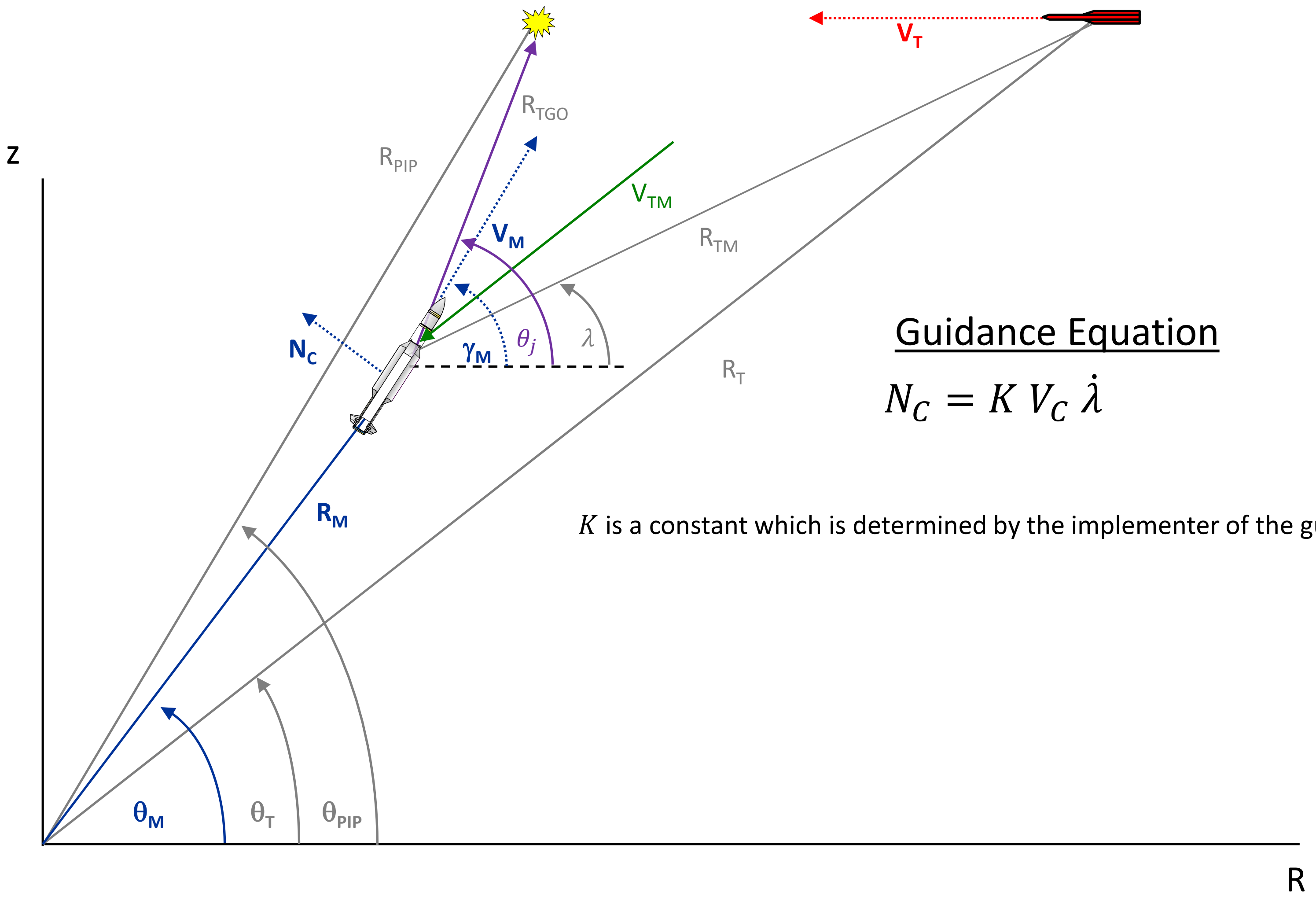
Synopsis

- Also known as line-of-sight rate guidance
- The word "navigation" is a bit of a misnomer as the missile navigation system is no more important in this guidance law than in any other we've discussed
- The missile is guided to intercept by that attempting to force the line-of-sight rate ($\dot{\lambda}$) is zero
  - Concept is similar to constant bearing guidance
  - Implementation makes PN more robust
    - Constant bearing guidance continuously computes a new "desired bearing" through inspection of the intercept triangle
    - PN does not require an intercept triangle – it monitors $\dot{\lambda}$
- The most robust of of the intuitive guidance laws
  - Used in many modern missiles as the terminal guidance law

It Will Be Shown that PN Also is a Modern Guidance Law

Guidance Law

Proportional Navigation



Guidance Equation

$$N_C = K\, V_C\, \dot{\lambda}$$

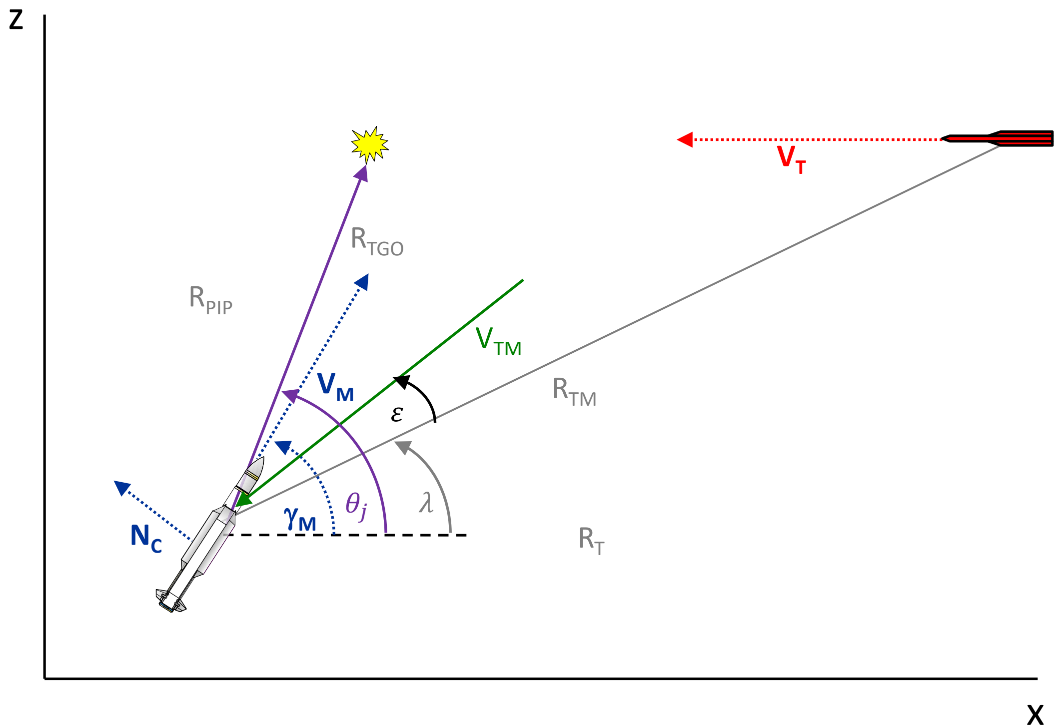
K is a constant which is determined by the implementer of the guidance law

Midcourse vs Terminal PN

- The guidance laws discussed so far were homing guidance laws
  - Proportional navigation can also be used to guide missiles to intercept points
  - This is called Midcourse PN Guidance, or PN to an Intercept Point
- Terminal PN is designed considering only the current missile and target states
- Midcourse PN is designed considering only the current missile states and the predicted intercept point
- The subtle difference between the two laws has considerable implications

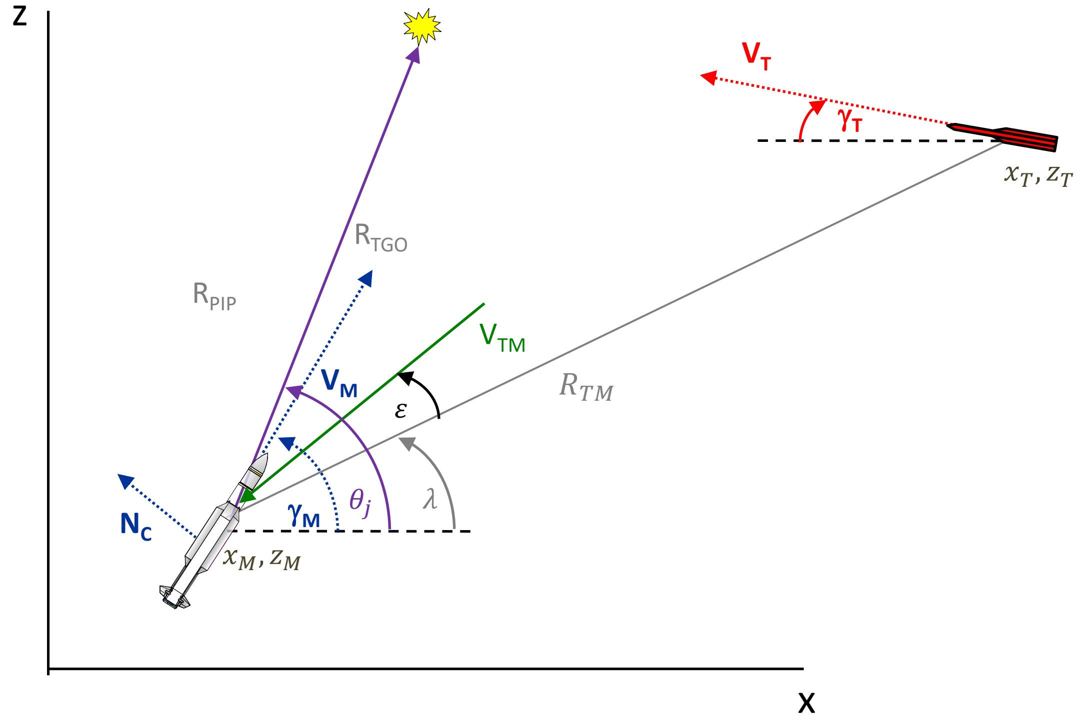
Geometry for Terminal Guidance
Proportional Navigation

- Due to the importance of the proportional navigation guidance law in both classic and modern guidance, some time will be spent deriving the original proportional navigation guidance law (referred to as true PN, or TPN, in literature)



Proportional Navigation

Derivation (Part I)



The following relations are deduced from the illustration

$$\Delta x = \Delta x_T - \Delta x_M$$

$$\Delta z = \Delta z_T - \Delta z_M$$

$$R_{TM} = \sqrt{\Delta x^2 + \Delta z^2}$$

$$\frac{\Delta x}{R} = \cos(\lambda)$$

$$\frac{\Delta z}{R} = \sin(\lambda)$$

$$\tan(\lambda) = \frac{\Delta z}{\Delta x}$$

Proportional Navigation

Derivation (Part II)

Notes

- Having defined the engagement geometry, the rate of change of various parameters is calculated
  - Line-of-sight rate ($\dot{\lambda}$)
  - Relative x rate ($\Delta\dot{x}$)
  - Relative z rate ($\Delta\dot{z}$)
- These parameters (combined) provide the basis for the equations of motion for the system
- However, PN-4 and PN-5 have trigonometry functions which make mathematical operations cumbersome
- Thus, small angle approximations are introduced

$$\cos(\lambda) \approx 1$$
$$\sin(\lambda) \approx \lambda$$

Take the time derivative of $\tan(\lambda)$, Δx, and Δz

(Eq. PN-1) $$\frac{d}{dt}[\tan(\lambda)] = \frac{\dot{\lambda}}{\cos^2(\lambda)} = \frac{\Delta\dot{z}\Delta x - \Delta z\,\Delta\dot{x}}{\Delta x^2}$$

(Eq. PN-2) $$\Delta\dot{x} = \dot{x}_T - \dot{x}_M = -V_T\cos(\gamma_T) - V_M\cos(\gamma_M)$$

(Eq. PN-3) $$\Delta\dot{z} = \dot{z}_T - \dot{z}_M = V_M\sin(\gamma_M) - V_T\sin(\gamma_T)$$

Substitute Eq. PN-2 and PN-3 into Eq. PN-1

(Eq. PN-4) $$R_{TM}\,\dot{\lambda} = V_M\sin(\gamma_M + \lambda) - V_T\sin(\gamma_T - \lambda)$$

(Eq. PN-5) $$\dot{\lambda} = -V_M\cos(\gamma_M + \lambda) - V_T\cos(\gamma_T - \lambda)$$

The linearize the equations by assuming small angle approximations

(Eq. PN-6) $$R_{TM}\,\dot{\lambda} \cong V_M(\gamma_M + \lambda) - V_T(\gamma_T - \lambda)$$

(Eq. PN-7) $$R_{TM}\,\dot{\lambda} \cong V_M\lambda_M - V_T\gamma_T + \lambda\,(V_M + V_T)$$

Proportional Navigation

Derivation (Part III)

Notes

- For semi-active seekers
  - The relative velocity (V_{TM}) is unknown
  - The relative range rate ($\dot{R}_{TM}$) is measured making it a very convenient rate of change parameter to use in terminal guidance computations
- Range rate can be computed as a function of V_{TM} and R_{TM} and the angle between the two vectors, ε
  - $\dot{R}_{TM} = \vec{V}_{TM} \cdot \hat{R}_{TM} = \cos(\varepsilon)\,|V_{TM}|$
- The small angle approximation used in the computation above allows for the definition of $\dot{R}_{TM}$ used in the derivation
- The closing speed (V_C) is used in conjunction with time-to-go when computing guidance metrics

Compute range rate, $\dot{R}_{TM}$

(Eq. PN-8) $\dot{R}_{TM} \approx -(V_T + V_M) = -V_C$

Define R_{TM} as function of time to go (T) using V_C

(Eq. PN-9) $R = V_C\,(T_0 - t) = V_C\,T$

Where:

T_0 is the initial time-to-go
t is the current time
T is the current time-to-go

Having defined all the required terms, we can compute the equation of motion by taking the derivative of PN-7

(Eq. PN-7) $R\,\dot{\lambda} \cong V_M\lambda_M - V_T\gamma_T + \lambda\,(V_M + V_T)$

(Eq. PN-10) $\frac{d}{dt}\left[R\,\dot{\lambda} \cong V_M\lambda_M - V_T\gamma_T + \lambda\,V_C\right]$

Proportional Navigation

Derivation (Part IV)

Notes

- The initial problem assumed that the target flight path angle (γ_T) was constant
  - No target maneuvers
  - $\dot{\gamma}_T = 0$
- The missile and target speeds are constant, which means their respective derivative are zero
  - $\dot{V}_M = 0$
  - $\dot{V}_T = 0$
- The proportionality constant "K" used in Eq. PN-13 has tremendous meaning
  - It is a key design parameter in the PN guidance law
  - It is also an important term in the optimal control laws of modern guidance

The resultant equation can be simplified through the relation: $\dot{R}_{TM} = -V_C$

(Eq. PN-11) $\dot{R}\,\dot{\lambda} + R\,\ddot{\lambda} = V_M\,\dot{\gamma}_M + V_C\,\dot{\lambda}$

(Eq. PN-12) $V_C\,T\,\ddot{\lambda} - 2\,V_C\,\dot{\lambda} = V_M\,\dot{\gamma}_M$

At this point, there is one equation and two unknowns ($\dot{\gamma}_M$ and $\dot{\lambda}$)

In order to solve the problem, a leap of faith is made that $\dot{\gamma}_M$ is proportional to $\dot{\lambda}$

(Eq. PN-13) $V_M\,\dot{\gamma}_M = -K\,V_C\,\dot{\lambda}$

Proportional Navigation

Derivation (Part V)

Notes

- The change of variable from t to T allows for us to solve the differential equation.
- Some rules to that are important during a change of variable
  - $\frac{d}{dt}[f] = -\frac{d}{dT}[f]$
  - $\frac{d^2}{dt^2}[f] = \frac{d^2}{dT^2}[f]$
- Furthermore, we'll use a shorthand notation as follows:
  - $\frac{d}{dt}[f] = \dot{f}$
  - $\frac{d}{dT}[f] = f'$

Substituting PN-13 into PN-12 and simplifying provides the equation of motion for the system

(Eq. PN-14) $\ddot{\lambda}\, T + (K-2)\dot{\lambda} = 0$

A change of independent variable from running time, t, to time-to-go, T readily provides a solution for $\dot{\lambda}$ (or λ')

(Eq. PN-15) $\lambda'' T - (K-2)\lambda' = 0$

(Eq. PN-16) $\lambda' = \lambda'_0 \left(\frac{T}{T_0}\right)^{K-2}, \ \dot{\lambda} = \dot{\lambda}_0 \left(\frac{T}{T_0}\right)^{K-2}$

Proportional Navigation

Derivation (Part VI)

Notes

- Since the missile can only attempt to achieve acceleration perpendicular to its velocity vector, we use the orientation of the velocity vector to describe components of acceleration
  - Along the velocity vector: $a_{\parallel v}$
  - Perpendicular to the velocity vector: $a_{\perp v}$
- It can be shown that through the laws of circular motion that
  - $a_{\perp v} = V\,\dot{\gamma}$

The acceleration perpendicular to the missile velocity vector, as indicated by the notes is

(Eq. PN-17) $$a_{\perp v} = V_M\,\dot{\gamma}$$

Substituting PN-13 and PN-16 into PN-17 gives the closed form solution for a proportional navigation guidance command

(Eq. PN-18) $$a_{\perp v} = -K\,V_C\,\dot{\lambda}_0\left(\frac{T}{T_0}\right)^{K-2}$$

At any instant, the acceleration can be computed by setting $T = T_0$

(Eq. PN-19) $$a_{\perp v} = -K\,V_C\,\dot{\lambda}_0$$

History and Variations

Proportional Navigation

- Proportional navigation guidance was developed with the assumption that $\dot{\gamma} \propto \dot{\lambda}$
  - Later it was proved (L.C. Yuan, 1948) that $\dot{\gamma} \propto \dot{\lambda}$
- The most efficient implementation of PN is not realizable by fin controlled missiles
  - A fin controlled missile can only achieve an acceleration perpendicular to its velocity vector
  - The optimum implementation of PN would require the acceleration command to be perpendicular to the line-of-sight vector
- Variations of PN include (but certainly aren’t limited to)
  - Pure PN (PPN)
  - Augmented PN (APN)

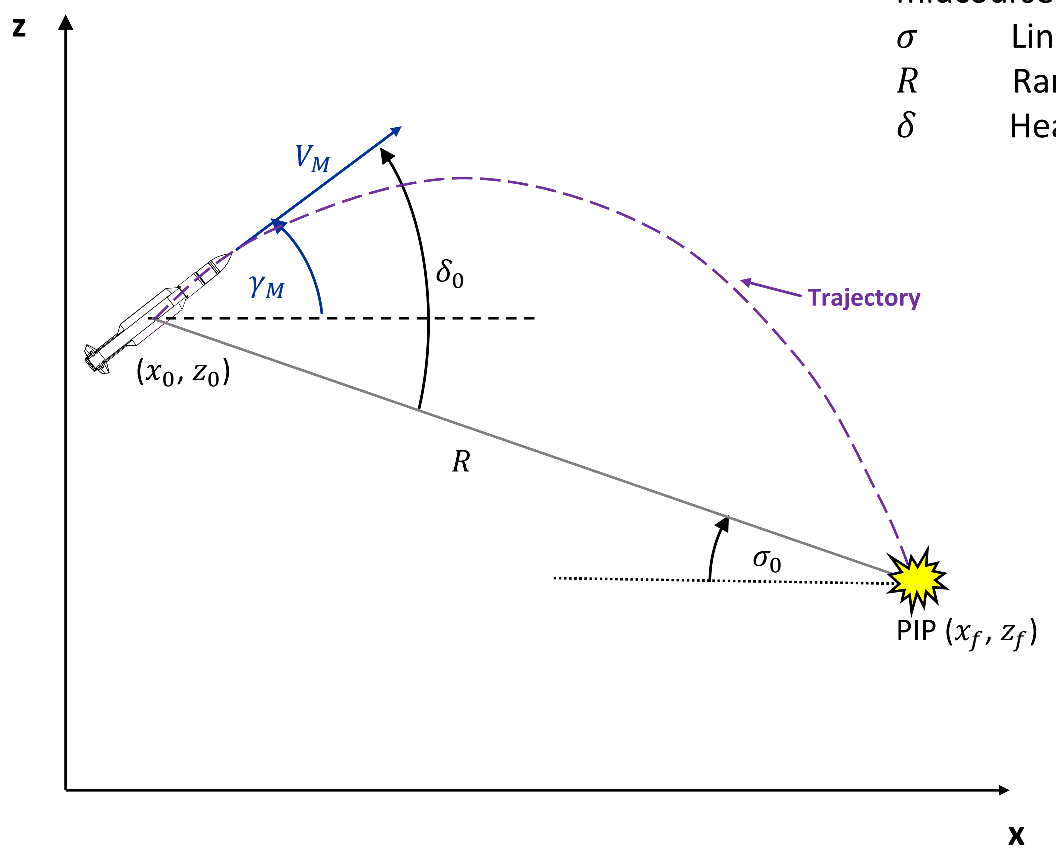
Midcourse PN Guidance

Synopsis

- Missile flies to an intercept point which is computed based upon an assumed target flight path and a known missile kinematic profile
  - Allows for the assumption of non-linear target trajectories
- Can be command or inertial guidance
- Assumptions
  - Intercept point is fixed
  - Missile speed is constant
- Drawbacks
  - More complicated to implement than previous guidance laws as intercept prediction must be considered
  - Prediction of intercept point will not be discussed at this time

Midcourse Guidance

Derivation (Part I)



From the illustration, we define some new terms as an attempt to differentiate between terminal PN and midcourse PN geometry

σ Line of sight to the intercept point

R Range from the missile to the intercept point

δ Heading error to the intercept point

Also from the illustration, we can define the following:

MPN-1 $x = x_f - R\cos(\sigma)$

MPN-2 $z = z_f + R\sin(\sigma)$

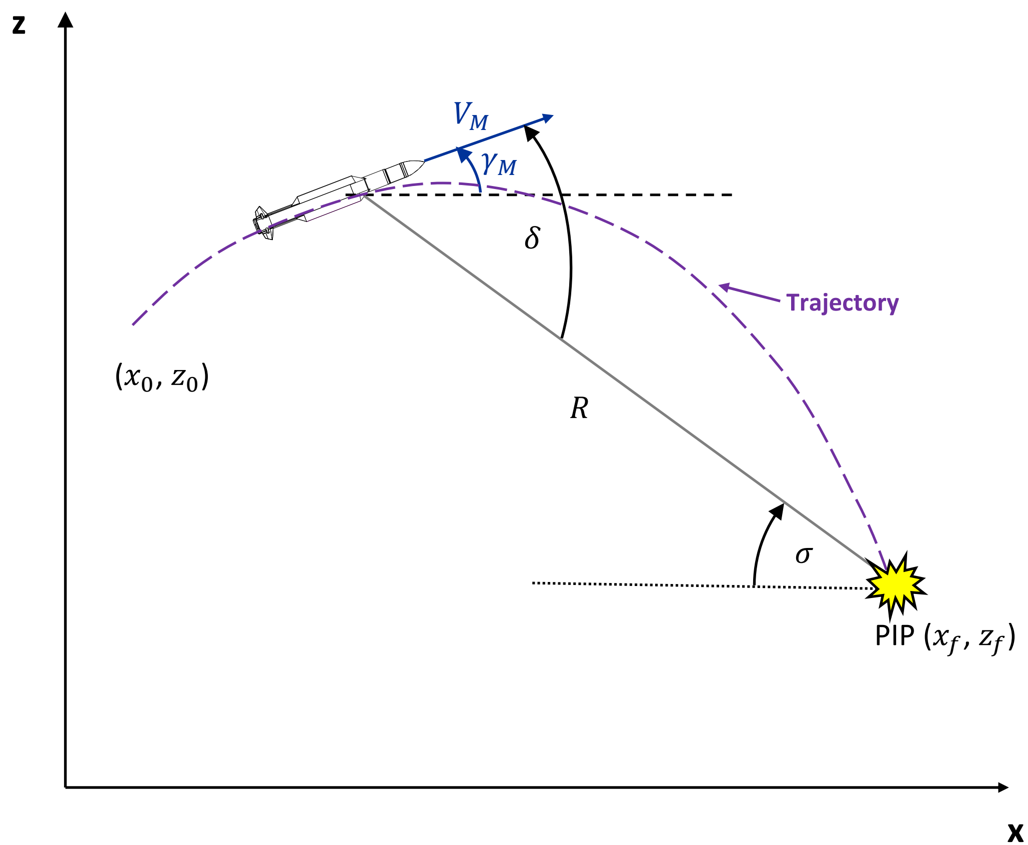
MPN-3 $\dot{x} = V_M\cos(\gamma_M)$

MPN-4 $\dot{z} = V_M\sin(\gamma_M)$

MPN-5 $\delta = \gamma + \sigma$

Midcourse Guidance

Derivation (Part II)



Taking the derivative of x and z give us

MPN-6 $\dot{x} = -\dot{R}\cos(\sigma) + R\,\dot{\sigma}\sin(\sigma)$

MPN-7 $\dot{z} = \quad \dot{R}\sin(\sigma) + R\,\dot{\sigma}\cos(\sigma)$

Combining MPN-6 and MPN-7 with MPN-3 and MPN-4 gives the following:

MPN-8 $V_M\cos(\gamma_M) = -\dot{R}\cos(\sigma) + R\,\dot{\sigma}\sin(\sigma)$

MPN-9 $V_M\sin(\gamma_M) = \quad \dot{R}\sin(\sigma) + R\,\dot{\sigma}\cos(\sigma)$

MPN-8 and MPN-9 are used to solve for $\dot{R}$ and $R\,\dot{\sigma}$, then small angle approximations are applied :

MPN-10 $\dot{R} = -V_M\cos(\gamma_M + \sigma)$

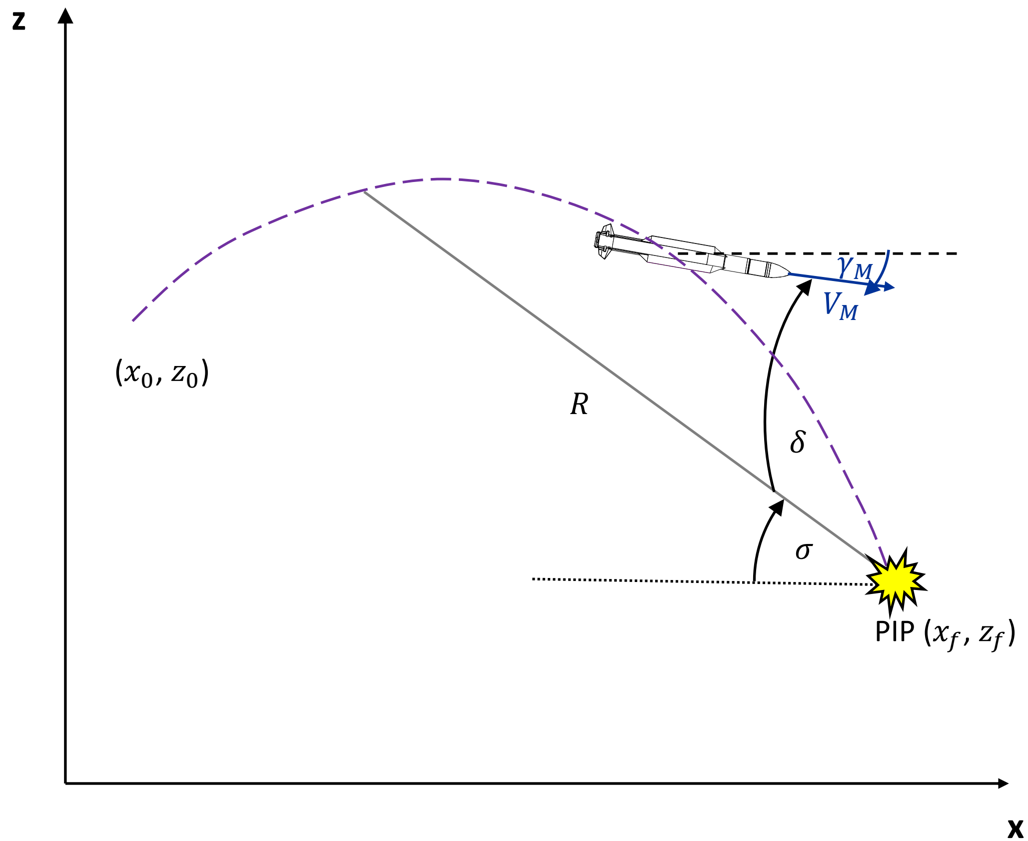
MPN-11 $R\,\dot{\sigma} = V_M\sin(\gamma_M + \sigma)$

MPN-12 $\dot{R} \cong -V_M$

MPN-13 $R\,\dot{\sigma} \cong V_M(\gamma_M + \sigma)$

Midcourse Guidance

Derivation (Part III)



The derivative of MPN-12 and MPN-13 is taken in order to determine the equations of motion

MPN-14 $\ddot{R} = 0$

MPN-15 $\dot{R}\dot{\sigma} + R\ddot{\sigma} = V_M(\dot{\gamma}_M + \dot{\sigma})$

Similar to the step used in deriving terminal PN, the following relationships are defined:

MPN-16 $R = V_M\,(T_0 - t)$

MPN-17 $R = R_0 - V_M t = R_0\,(\frac{T}{T_0})$

Substituting MPN-12 and MPN-16 into MPN-15 gives us the equation of motion for the system

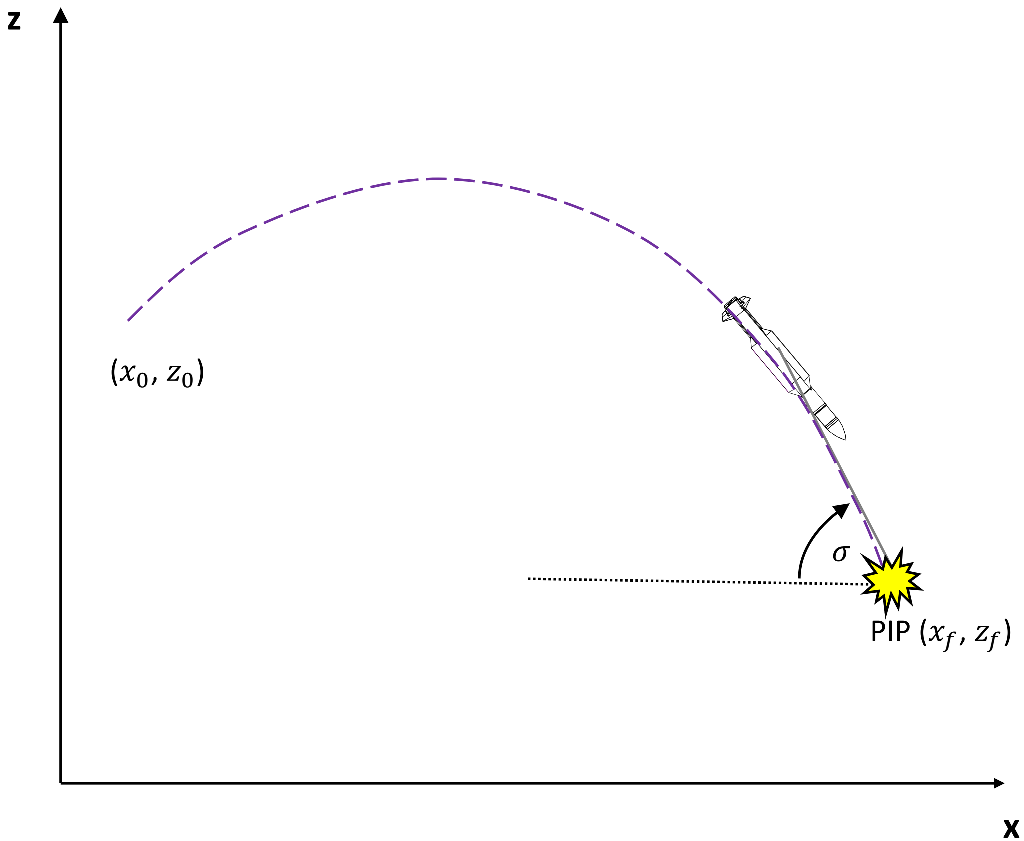
MPN-18 $-V_M\,\dot{\sigma} + V_M(T_0 - t)\ddot{\sigma} = V_M(\dot{\gamma}_M + \dot{\sigma})$

MPN-19 $\dot{\sigma} + (T_0 - t)\ddot{\sigma} = (\dot{\gamma}_M + \dot{\sigma})$

MPN-20 $\dot{\sigma} + T\,\ddot{\sigma} = (\dot{\gamma}_M + \dot{\sigma})$

Midcourse Guidance

Derivation (Part IV)



Once again, with two equations and two unknowns, a proportional relationship is assumed between $\dot{\gamma}$ and a line-of-sight rate

MPN-21 $\dot{\gamma}_M = -K\,\dot{\sigma}$

Thus, MPN-20 becomes this

MPN-22 $T\,\ddot{\sigma} - (K-2)\dot{\sigma} = 0$

And similar to before, our guidance law is shown to be:

MPN-23 $N_C = -K\,V_M \sigma_0' \left(\frac{T}{T_0}\right)^{K-2}$

Using equation MPN-5 and MPN-13 in conjunction with MPN-23 provides an alternative form of the guidance command which is more intuitive for midcourse guidance

MPN-23 $N_C = -K\,\frac{V_M}{T}\,\delta_0 \left(\frac{T}{T_0}\right)^{K-2}$

Comparison of Midcourse and Terminal PN

- The midcourse and terminal PN guidance laws are very similar
  - Both rely upon line-of-sight rate being proportional to flight path angle
  - Magnitude of the commands can be shown to be equal
- There is one very important difference between the guidance laws is the direction in which the command is applied
  - Midcourse PN applies the acceleration commands perpendicular to the missile velocity vector
  - Terminal PN applies the acceleration commands perpendicular to the line-of-sight
- Midcourse PN is the more efficient guidance law for fin controlled interceptors as the interceptor is able to apply all the commanded acceleration in a direction in which it can be achieved

References

Lockheed Martin Material used as guide for this lecture (topics to cover), etc.

1. Luk-Paszyc, J.W. *Missile Flight Control Systems – Autopilots*. Missile System Engineering Fundamentals. Lockheed Martin course (~1984)

2. Corse, J.T. Midcourse Guidance Course. Lockheed Martin summer course, 1998

There are a plethora of books that cover guidance, navigation, and control. Suggested books for those wanting more detail with regard to guidance, navigation, and control are:

1. Shneydor, N. A., *Missile Guidance and Pursuit: Kinematics, Dynamics and Control*.

2. Siouris, George, *Missile Guidance and Control Systems*.