Simulating Simple Electrostatic Capacitors

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Abstract—**This lab applies Faraday's Law of Induction, which states that a changing magnetic field induces an EMF voltage in a conductor. In this specific case, the changing magnetic field was supplied by a permanent magnet with a constant, non-zero velocity moving through the center of a copper coil. The ends of the coil were connected through a resistor, which allowed the induced voltage to create a current by Ohm's law.**

I. Introduction & Setup

This lab started as far from the simulation software as possible, locally in FreeCAD. While it is possible (and encouraged for most) to model a system directly in the simulation software, it is finicky at best. Furthermore, the documentation for it is almost as proprietary as the software itself. For this reason, it made sense to use an external tool for design purposes. The technical drawings are included after the appendix.

All components were exported from FreeCAD into the simulation software, positioned properly and the simulation was set up. Currents were measured at the ends of the coil, and the magnet was moved along a path starting below the coil and ending above it. A 100Ω resistor was placed in series with the coil, so current could be measured through the coil.

The magnet started 2[mm] below the coil and traveled upwards at 500[mm/s] for 10[ms]. The speed was initially 10[mm/s], but this needed to be increased as the magnet only travelled 0.1[mm]. This was so slow that the magnet did not even reach the bottom of the coil before the simulation ended. 500[mm/s] seemed like the most reasonable speed since it would travel a full 5[mm] over the course of the simulation. At this speed, the magnet travels from -2[mm] to +3[mm], which is above the top of the coil.

The importance of it travelling through the coil is that the magnetic field is strongest close to the magnet, so it is critical to get the magnet as close as possible to the coil to see the best induction effects. The induced currents are in accordance with Faraday's Law of Induction seen in equation 1. For a coil of wire, the effects of Faraday's law are multiplied scaled up with each winding. The voltage across a coil with N windings and magnetic flux, Φ_B, through the center is given by equation 2. Therefore, by Ohm's Law, the current through the inductor is just the voltage given by equation 2 divided by 100Ω.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{1}$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} \tag{2}$$

II. Results & Analysis

Passing the magnet through the coil creates a changing magnetic field from the coil's perspective. From the magnet's perspective field is static, and the coil is moving towards it. Therefore, when the animation is played, the magnetic field appears to move upwards at the same rate as the magnet. The shape of the magnetic field around the magnet, passing through the coils is seen in figure 1. The magnetic field is originating from the magnet, curling back on itself, and then terminating back at the magnet. This makes sense, since all magnets have a north and south pole, with the magnetic field lines travelling between them.
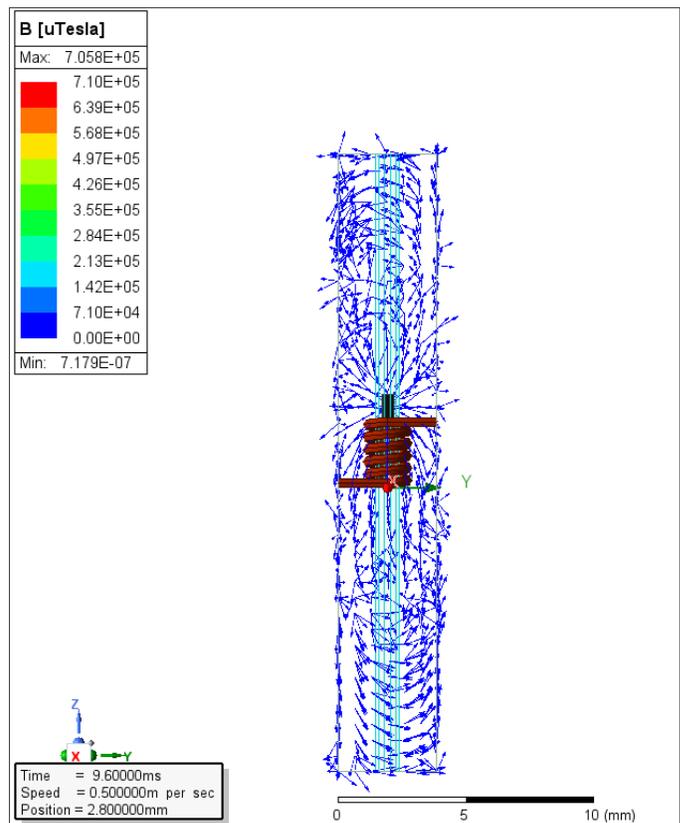

Fig. 1. Magnetic field vectors in the region

As seen in figure 2, there are two spikes in current. As the magnet approaches the coil, it makes sense that the magnetic field increases rapidly. It also makes sense, that as the magnet leaves the coil, around 9[ms] or so, the current will decrease. However the dropout in the middle, may be unexpected. Faraday's Law of Induction relates voltage to the time rate of change of the magnetic field, and not directly to its strength. Following this line of thought, it must be that the most change in magnetic field strength happens when the magnet is either entering or leaving the coil.

Fig. 2. Current through the coil over time

III. Conclusion

The drop in current while the magnet is in the coil was quite unexpected at first, but after recalling Faraday's Law of Induction, the results made more sense. Since the change in magnetic field is highest when the magnet is approaching or receding from the coil, it makes sense that spikes in current occur here. However, when the magnet is centered in the coil, the magnetic field is symmetric with respect to the coil. For this reason, magnetic field changes very little at this point, and in a perfectly symmetric system, the current would probably go to zero. Thinking back to one dimensional kinematics, at the apex of a projectile being thrown straight into the air, the instantaneous velocity goes to zero.

Measuring the time rate of change of the magnetic flux is analogous to projectile motion in this way. Think about it as the magnetic field strength reaching a maximum in the coil before decreasing and going to zero at $t = \infty$. Since the strength reaches a maximum, the rate of change of strength is by definition, zero.

The voltage across the coil can be recovered since the resistor has a known value of 100Ω. Simply by scaling the current results by a factor of 100 will yield the EMF voltage of the coil. Finally, by working backwards from Faraday's Law of Induction, since the number of coils is known to be $N = 5$, by taking the negative of the integral of the EMF voltage with respect to time, the magnetic flux through the coil can be extrapolated.

By knowing, the current, voltage, and magnetic flux of the coil with respect to time, much is learned about the system as a whole, and even more information, such as inductance characteristics, can be derived.