Mathematica
Project 1

1. Consider parametric equations given by $x = 3\cos(t/3) - \cos t$ and $y = 3\sin(t/3) - \sin t$.

a) Graph the curve represented by the parametric equations above.

b) Find the slope of the line tangent to the curve at the point where $t = \pi/4$.

c) Find the arc length of the curve from $t = 0$ to $t = 3\pi/2$.

2. Consider the rose curve $r = \cos(2\theta)$ for $-2\pi \le \theta \le 2\pi$.

a) Plot its graph.

b) Find the area of one petal of the curve.

3. Graph and find the area of the common interior of $r = 3 - 2\sin\theta$ and $r = -3 + 2\sin\theta$.

4. Find the length of the given curve on the specified interval.

a) $r = 1 + \sin\theta$, $0 \le \theta \le 2\pi$.

b) $r = 6 \times (1 + \cos\theta)$, $0 \le \theta \le 2\pi$.

5. Consider the polar equations $r = 4\sin\theta$ and $r = 2 \times (2 - (\sin\theta)\text{^}2)$

a) Graph the polar equations on the same axes.

b) Find the points of intersection of the curves.

c) Find the circumference of each curve.

8. Let $\mathbf{u} = \langle 2, 2, 1\rangle$, $\boldsymbol{v} = \langle 1, -2, 2\rangle$, and $\boldsymbol{w} = \langle 1, 3, 2\rangle$. Find

a) the length of $2\mathbf{u}-3\mathbf{v}$

b) the dot product of **v** and **w**

c) the cross product of **u** and **w**

d) the area of the parallelogram spanned by **v** and **w**.

e) the volume of the parallelepiped spanned by **u**, **v**, and **w.**

6. Graph $\mathbf{r}(t)$:

a) $\mathbf{r}(t) = \langle \cos(2t), \cos t, \sin t\rangle$

b) $\mathbf{r}(t) = \left\langle t + 15, e^{0.08t}\cos t, e^{0.08t}\sin t\right\rangle$

7. Evaluate the limits:

a) $\lim_{t\to\pi}\langle \sin 2t, \cos t, \tan 4t\rangle$

b) $\lim_{t\to 0}\left\langle \frac{1}{t+1}, \frac{e^t-1}{t}, 4t\right\rangle$

8. Compute the derivative and integral:

a) $\mathbf{r}(t) = \langle \tan t, 4t - 2, \sin t\rangle$

b) $\mathbf{r}(t) = \left\langle e^t, e^{2t}\right\rangle$

9. Compute the length of curve over the given interval:

a) $\mathbf{r}(t) = \langle 2\sin t, 6t, 2\cos t\rangle$, $-6 \le t \le 6$ b) $\mathbf{r}(t) = \left\langle 12t, 8t^{3/2}, 3t^2\right\rangle$, $0 \le t \le 1$

10. The *Cornu spiral i*s defined by $\mathbf{r}(t) = \langle x(t), y(t)\rangle$, where $x(t) = \int_0^t \sin\left(\frac{u^2}{2}\right) du$ and $y(t) = \int_0^t \cos\left(\frac{u^2}{2}\right) du$.

a) Plot the Cornu spiral over various intervals for t.

b) Find a formula for its arc length along the interval $-a \le t \le a$, where a is a positive real number.

c) What is its arc length in the limit as $a \to \infty$?

11. Find a formula for the curvature of the general helix $\mathbf{r}(t) = a \cos t\, \mathbf{i} + a \sin t\, \mathbf{j} + c\, t\, \mathbf{k}$.

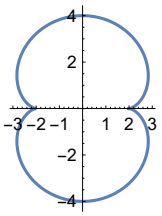
12. Calculate the velocity and acceleration vectors and the speed if $\mathbf{r}(t) = \cos t\, \mathbf{i} + \sin t\, \mathbf{j} + \tan(2t)\, \mathbf{k}$, $t = \frac{\pi}{6}$.

13. Find $\mathbf{r}(t)$ and $\mathbf{v}(t)$ given that $\mathbf{a}(t) = e^{3t}\, \mathbf{i} + 4t\, \mathbf{j} + (t-2)\, \mathbf{k}$, $\mathbf{v}(0) = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{r}(0) = 0\, \mathbf{i} + 3\, \mathbf{j} + 4\, \mathbf{k}$.

Question 1.

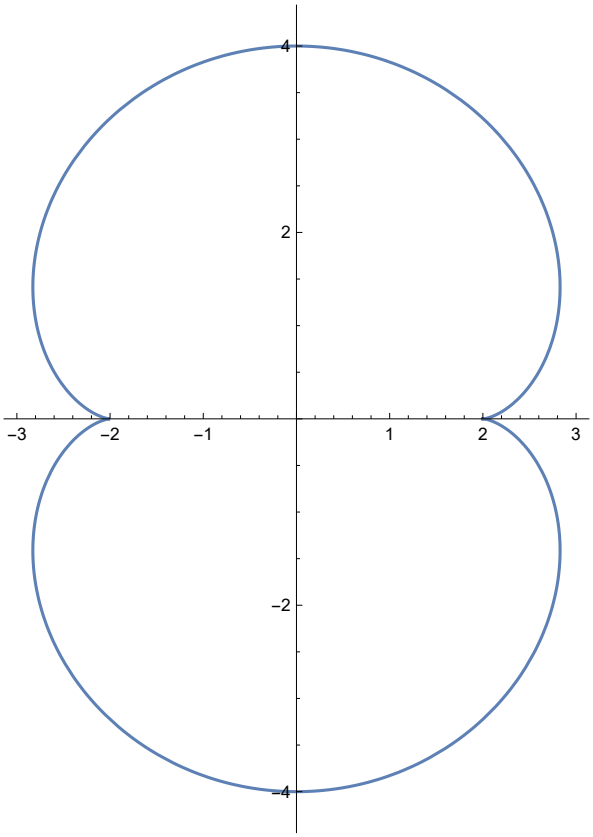
In[ ]:=
```
ParametricPlot[{3 Cos[t / 3] - Cos[t], 3 Sin[t / 3] - Sin[t]}, {t, 0, 6 Pi}]
```

In[ ]:=




```
D[3 Sin[t / 3] - Sin[t]] / D[3 Cos[t / 3] - Cos[t], t]
```

Out[ ]=



In[•]:=
$$\frac{3\,\text{Sin}\left[\frac{t}{3}\right]-\text{Sin}[t]}{-\text{Sin}\left[\frac{t}{3}\right]+\text{Sin}[t]}$$

```
t = Pi / 4
```

$$\frac{3\,\text{Sin}\left[\frac{t}{3}\right]-\text{Sin}[t]}{-\text{Sin}\left[\frac{t}{3}\right]+\text{Sin}[t]}$$

Out[•]=
$$\frac{3\,\text{Sin}\left[\frac{t}{3}\right]-\text{Sin}[t]}{-\text{Sin}\left[\frac{t}{3}\right]+\text{Sin}[t]}$$

Out[•]=
$$\frac{\pi}{4}$$

Out[•]=
$$\frac{-\frac{1}{\sqrt{2}}+\frac{3\times\left(-1+\sqrt{3}\right)}{2\sqrt{2}}}{\frac{1}{\sqrt{2}}-\frac{-1+\sqrt{3}}{2\sqrt{2}}}$$

In[•]:=
```
t =.
x[t_] = 3 Cos[t / 3] - Cos[t]
y[t_] = 3 Sin[t / 3] - Sin[t]
Integrate[Sqrt[x'[t]^2 + y'[t]^2], {t, 0, 3 Pi / 2}]
```

Out[•]=
$$3\,\text{Cos}\left[\frac{t}{3}\right]-\text{Cos}[t]$$

Out[•]=
$$3\,\text{Sin}\left[\frac{t}{3}\right]-\text{Sin}[t]$$

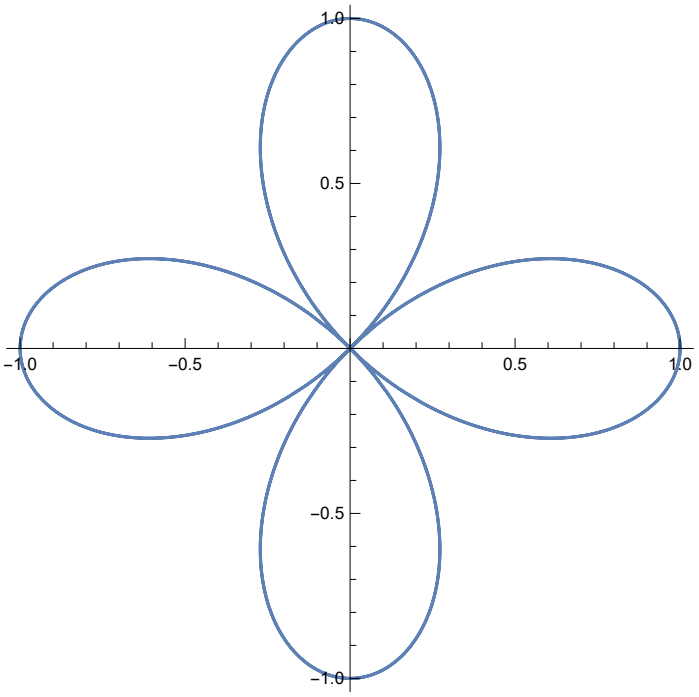
Out[•]=
$$6$$

Question 2.

In[ ]:=
```
r[t_] = Cos[2 t]
PolarPlot[r[t], {t, -2 Pi, 2 Pi}]
```

Out[ ]=
$$\cos[2t]$$

Out[ ]=



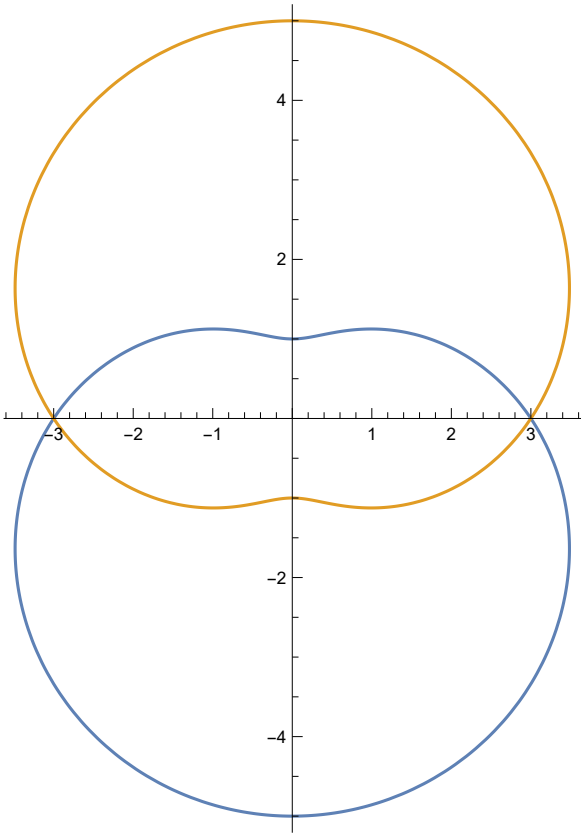
In[ ]:=
```
Integrate[r[t]^2, {t, 0, Pi / 2}] / 2
```

Out[ ]=
$$\frac{\pi}{8}$$

Question 3.

In[ ]:=
```
r1[t_] := 3 - 2 Sin[t]
r2[t_] := 2 Sin[t] - 3
PolarPlot[{r1[t], r2[t]}, {t, 0, 2 Pi}]
```

Out[ ]=



In[ ]:=
```
Integrate[r1[t]^2, {t, 0, Pi}]
```

Out[ ]= $-24+11\pi$

Question 4.

In[ ]:=
```
r1[t_] := 1 + Sin[t]
r2[t_] := 6 × (1 + Cos[t])
Integrate[Sqrt[1 + r1'[t]^2], {t, 0, 2 Pi}]
Integrate[Sqrt[1 + r2'[t]^2], {t, 0, 2 Pi}]
```

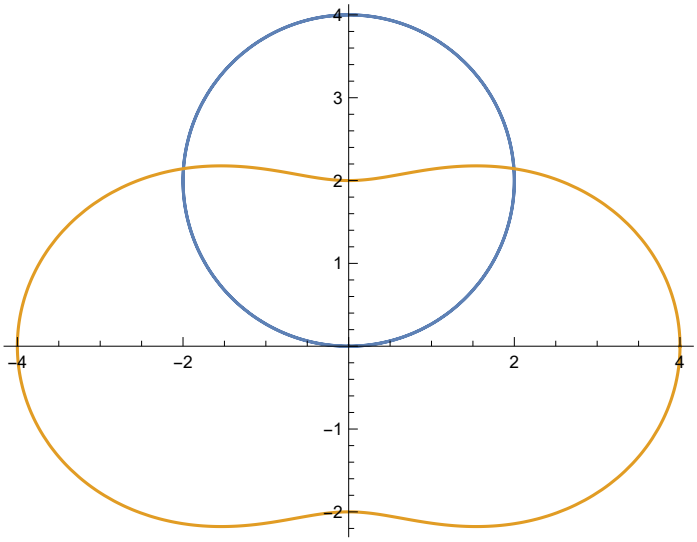
Out[ ]= $4\sqrt{2}\ \text{EllipticE}\left[\frac{1}{2}\right]$

Out[ ]= $4\,\text{EllipticE}[-36]$

Question 5.

In[1]:=
```
r1[t_] := 4 Sin[t]
r2[t_] := 2 × (2 - Sin[t]^2)
PolarPlot[{r1[t], r2[t]}, {t, 0, 2 Pi}]
```

Out[3]=



In[4]:=
```
Solve[r1[t] == r2[t], t]
```

Out[4]=
$$\left\{\left\{t \to -\text{ArcSin}\left[1-\sqrt{3}\right]+2\pi c_1 \text{ if } c_1 \in \mathbb{Z}\right\}, \left\{t \to \pi+\text{ArcSin}\left[1-\sqrt{3}\right]+2\pi c_1 \text{ if } c_1 \in \mathbb{Z}\right\},\right.$$
$$\left.\left\{t \to -\text{ArcSin}\left[1+\sqrt{3}\right]+2\pi c_1 \text{ if } c_1 \in \mathbb{Z}\right\}, \left\{t \to \pi+\text{ArcSin}\left[1+\sqrt{3}\right]+2\pi c_1 \text{ if } c_1 \in \mathbb{Z}\right\}\right\}$$

In[5]:=
```
Integrate[Sqrt[1 + r1'[t]^2], {t, 0, 2 Pi}]
Integrate[Sqrt[1 + r2'[t]^2], {t, 0, 2 Pi}]
```

Out[5]=
$$4\sqrt{17}\ \text{EllipticE}\left[\frac{16}{17}\right]$$

Out[6]=
$$4\ \text{EllipticE}[-4]$$

Question 8.

In[7]:=
```
u := {2, 2, 1}
v := {1, -2, 2}
w := {1, 3, 2}
Length[2 u - 3 v]
Dot[v, w]
Cross[u, w]
Length[Cross[v, w]]
Dot[u, Cross[v, w]]
```

Out[10]= 3

Out[11]= -1

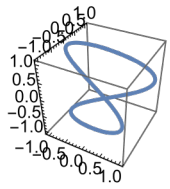
Out[12]= {1, -3, 4}

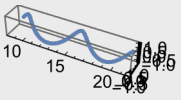
Out[13]= 3

Out[14]= -15

Question 6.

In[21]:=
```
ParametricPlot3D[{Cos[2 t], Cos[t], Sin[t]}, {t, -2 Pi, 2 Pi}]
ParametricPlot3D[{t + 15, (E^0.08) Cos[t], (E^0.08) Sin[t]}, {t, -2 Pi, 2 Pi}]
```





Question 7.

In[26]:=
```
Limit[{Sin[2 t], Cos[t], Tan[4 t]}, t → Pi]
```

Out[26]= {0, -1, 0}

In[27]:=
```
Limit[{1 / (1 + t), (E^t - 1) / t, 4 t}, t → 0]
```

Question 8.

In[31]:=
```
r1[t_] := {Tan[t], 4 t - 2, Sin[t]}
r2[t_] := {E^t, E^(2 t)}
r1'[t]
r2'[t]
```

Out[33]= $\{\text{Sec}[t]^2, 4, \text{Cos}[t]\}$

Out[34]= $\{e^t, 2\, e^{2\,t}\}$

Question 9.

In[35]:=
```
r1[t_] := {2 Sin[t], 6 t, 2 Cos[t]}
r2[t_] := {12 t, 8 t^(3 / 2), 3 t^2}
Integrate[Length[r1'[t]], {t, -6, 6}]
Integrate[Length[r2'[t]], {t, 0, 1}]
```

Out[37]= 36

Out[38]= 3

Question 10.

In[43]:=
```
x[t_] := Integrate[Sin[u^2 / 2], {u, 0, t}]
y[t_] := Integrate[Cos[u^2 / 2], {u, 0, t}]
r[t_] := {x[t], y[t]}
ParametricPlot[r[t], {t, 0, 2 Pi}]
```

Integrate: Invalid integration variable or limit(s) in {{2, 2, 1}, 0, 0.000128228}.

Integrate: Invalid integration variable or limit(s) in {{2, 2, 1}, 0, 0.000128228}.

NIntegrate: Tag List in {2, 2, 1} is Protected.

NIntegrate: Tag List in {2, 2, 1} is Protected.

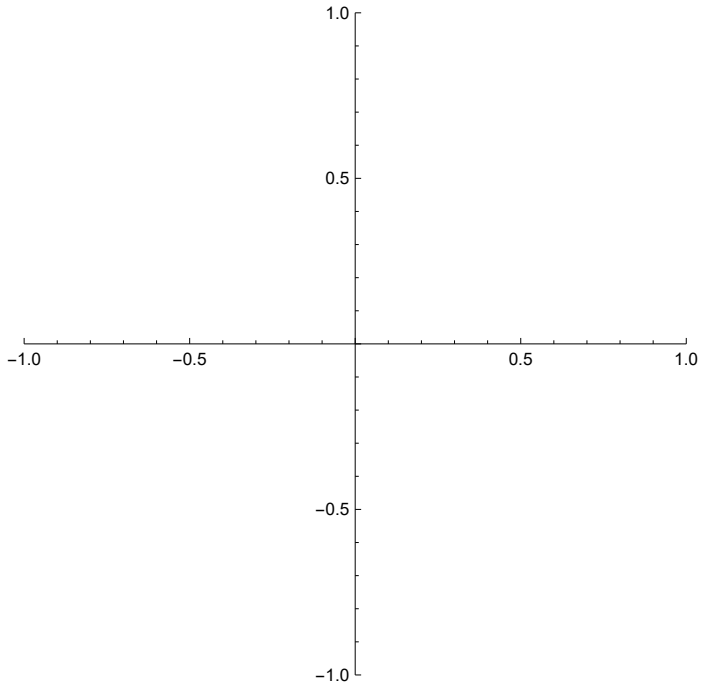
NIntegrate: Tag List in {2, 2, 1} is Protected.

General: Further output of NIntegrate::write will be suppressed during this calculation.

Integrate: Invalid integration variable or limit(s) in {{2, 2, 1}, 0, 0.128356}.

General: Further output of Integrate::ilim will be suppressed during this calculation.

Out[46]=



Question 11.

In[47]:=
```
r[t_] := {a * Cos[t], a * Sin[t], c * t}
Length[Cross[r'[t], r''[t]]] / (Length[r'[t]]^3)
```

Out[48]=
$$\frac{1}{9}$$

Question 12.

In[49]:=
```
r[t_] := {Cos[t], Sin[t], Tan[2 t]}
r'[Pi / 6]
r''[Pi / 6]
Length[r'[Pi / 6]]
```

Out[50]= $\left\{-\frac{1}{2}, \frac{\sqrt{3}}{2}, 8\right\}$

Out[51]= $\left\{-\frac{\sqrt{3}}{2}, -\frac{1}{2}, 32\sqrt{3}\right\}$

Out[52]= 3

Question 13

In[61]:=
```
a[t_] = {E^(3 t), 4 t, (t - 2)}
v[t_] = Integrate[a[t], t]
r[t_] = Integrate[v[t], t]
```

Out[61]= $\left\{e^{3t}, 4t, -2+t\right\}$

Set: Tag List in {1, –2, 2}[t_] is Protected.

Out[62]= $\left\{\frac{e^{3t}}{3}, 2t^2, -2t+\frac{t^2}{2}\right\}$

Out[63]= $\int \{1, -2, 2\}[t]\, dt$

Set: Tag List in {1, –2, 2}[t_] is Protected.