Rowan University

ECE Department

Signals and Systems

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Final

- Type your name and sign the statement on the answer sheet.
- Write your choice beside each item on the answer sheet.
- Only upload the answer sheet on canvas
- There are 15 problems on the final.
- The problems are not in order of difficulty. We recommend that you read through all the problems, then do the problems in whatever order suits you best.

Name

Problem 1

If $x(t) = u(t+2) - u(t-4)$ where $u(t)$ is the unit-step, an expression for $x(2t-2)$ is given by:

A. $u(t) - u(t-3)$

B. $u(t) - u(t-6)$

C. $u(t+2) - u(t-4)$

D. $u(t) - u(t-4)$

E. $u(t) - u(t-2)$

F. $u(t+2) - u(t-2)$

Problem 2

If $x(t)$ is even, then $x(5t)$ is

A. Even

B. Odd

C. Neither even nor odd

D. Both even and odd

Questions 3, 4 and 5 pertain to the following systems

$$\begin{aligned} y(t) = S_1[x(t)] &= \int_{t-10}^{t} \cos(\tau)x(\tau)d\tau \\ y(t) = S_2[x(t)] &= \cos(tx(t)) \\ y(t) = S_3[x(t)] &= t\cos(x(t)) \end{aligned}$$

Note that $x(t)$ is the input and $y(t)$ is the output.

Problem 3

Which of the systems is/are linear?

A. S_1 only

B. S_2 only

C. S_3 only

D. S_1 and S_2 only

E. S_1 and S_3 only

F. S_2 and S_3 only

G. S_1, S_2 and S_3

Problem 4

Which of the systems is/are causal?

A. S_1 only

B. S_2 only

C. S_3 only

D. S_1 and S_2 only

E. S_1 and S_3 only

F. S_2 and S_3 only

G. S_1, S_2 and S_3

Problem 5

Which of the systems is/are bounded-input bounded-output (BIBO) stable?

A. S_1 only

B. S_2 only

C. S_3 only

D. S_1 and S_2 only

E. S_1 and S_3 only

F. S_2 and S_3 only

G. S_1, S_2 and S_3

Problem 6

The impulse response of a linear, time-invariant (LTI) continuous-time system is

$$h(t) = u(t) - u(t-1),$$

where $u(t)$ is the unit step. If the input to this system is $x(t)$, the system output is

A.

$$y(t) = \int_{t-1}^{t} x(\tau)d\tau$$

B.

$$y(t) = \int_{t}^{t+1} x(\tau)d\tau$$

C.

$$y(t) = \int_{t-1}^{t+1} x(\tau)d\tau$$

Problem 7

What is the Laplace transform of $x(t) = \sin(\pi t)[u(t) - u(t-1)]$? Note that $u(t)$ is the unit-step.

A.

$$X(s) = \frac{\pi}{s^2 + \pi^2}$$

B.

$$X(s) = \frac{\pi e^{-s}}{s^2 + \pi^2}$$

C.

$$X(s) = \frac{\pi s e^{-s}}{s^2 + \pi^2}$$

D.

$$X(s) = \frac{\pi(1 + e^{-s})}{s^2 + \pi^2}$$

E.

$$X(s) = \frac{\pi(1 - e^{-s})}{s^2 + \pi^2}$$

Problem 8

The impulse response of a linear, time-invariant (LTI) continuous-time system is

$$h(t) = e^{-t}u(t),$$

where $u(t)$ is the unit step. If the output of the system is $y(t) = (1-e^{-t})u(t)$, the input to the system $x(t)$ is

A. $x(t) = \delta(t)$ where $\delta(t)$ is the impulse signal

B. $x(t) = u(t)$

C. $x(t) = e^{-t}u(t)$

D. $x(t) = (1 - e^{-t})u(t)$

Problem 9

A differential equation with initial condition $x(0^-) = 0$ is given by

$$\frac{dx}{dt} + 5x(t) = \delta(t) \tag{1}$$

where $\delta(t)$ is the impulse function. What is the solution to the differential equation? Note that $u(t)$ is the unit-step.

A. $x(t) = 1$

B. $x(t) = \delta(t)$

C. $x(t) = e^{-5t}u(t)$

D. $x(t) = [1 - e^{-5t}]u(t)$

Problem 10

Which of the following is the Laplace inverse of

$$X(s) = \frac{2s + 3}{s^2 + 4s + 13}$$

Note that $u(t)$ is the unit-step.

A.

$$x(t) = e^{2t}(2\cos(3t) - \frac{1}{3}\sin(3t))\ u(t)$$

B.

$$x(t) = e^{-2t}(2\cos(3t) - \frac{1}{3}\sin(3t))\ u(t)$$

C.

$$x(t) = (2\cos[3(t+2)] - \frac{1}{3}\sin[3(t+2)])\ u(t)$$

D.

$$x(t) = (2\cos[3(t-2)] - \frac{1}{3}\sin[3(t-2)])\ u(t)$$

Questions 11 and 12 pertain to the signal $x(t) = 8\cos(2t)$.

Problem 11

The exponential Fourier series of $x(t)$ is

$$\sum_{k=-\infty}^{\infty} X_k e^{jk\Omega_0 t} \tag{2}$$

where $\Omega_0 = 2\pi/T_0$ and T_0 is the period of $x(t)$. Find the value of

$$\sum_{k=-\infty}^{\infty} |X_k|^2 \tag{3}$$

A. 1

B. 2

C. 4

D. 8

E. 16

F. 32

Problem 12

Find the trigonometric Fourier series of $x(t - \pi)$.

A. $8[1 + \cos 2t + \sin 2t]$

B. $8[1 + \cos 2t]$

C. $8[1 + \sin 2t]$

D. $8[\cos 2t + \sin 2t]$

E. $8\cos 2t$

F. $8\sin 2t$

Problem 13

Consider the linear time-invariant system with a frequency response given by

$$H(j\Omega) = \frac{1}{1 + j\Omega/2} = \frac{2}{2 + j\Omega}$$

The input to the system is

$$x(t) = 2\cos(2t + \pi/2)$$

The output $y(t)$ of this system is given by

A. $y(t) = 2\cos(2t + \frac{\pi}{4})$

B. $y(t) = 2\cos(2t - \frac{\pi}{4})$

C. $y(t) = 2\cos(2t)$

D. $y(t) = 2\cos(2t + \frac{\pi}{2})$

E. $y(t) = 2\cos(2t - \frac{\pi}{2})$

F. $y(t) = \sqrt{2}\cos(2t + \frac{\pi}{4})$

G. $y(t) = \sqrt{2}\cos(2t - \frac{\pi}{4})$

H. $y(t) = \sqrt{2}\cos(2t)$

I. $y(t) = \sqrt{2}\cos(2t + \frac{\pi}{2})$

J. $y(t) = \sqrt{2}\cos(2t - \frac{\pi}{2})$

Problem 14

Which of the following is the Fourier transform of

$$x(t) = e^{-2t}\ u(t),$$

where $u(t)$ is the unit step function?

A. $\frac{1}{2+j\ \Omega}$

B. $\frac{1}{2-j\ \Omega}$

C. $2 + j\ \Omega$

D. $2 - j\ \Omega$

Problem 15

Consider a linear, time-invariant (LTI) system with an impulse response $h(t) = te^{-t}u(t)$ where $u(t)$ is the unit-step function. Consider the following statements.

(I) All poles of the system are at $s = -1$.

(II) The system is causal.

(III) The magnitude of the Fourier transform of $h(t)$ equals 1 at a frequency of 0 radians/second.

(IV) An exponential Fourier series of $h(t)$ can be calculated.

Which of the following statements is/are true?

A. I and II only

B. I, II and III only

C. II and IV only

D. I, II and IV only

E. III and IV only

F. II, III and IV only

G. All four statements I, II, III and IV are true.