Trajectory Design

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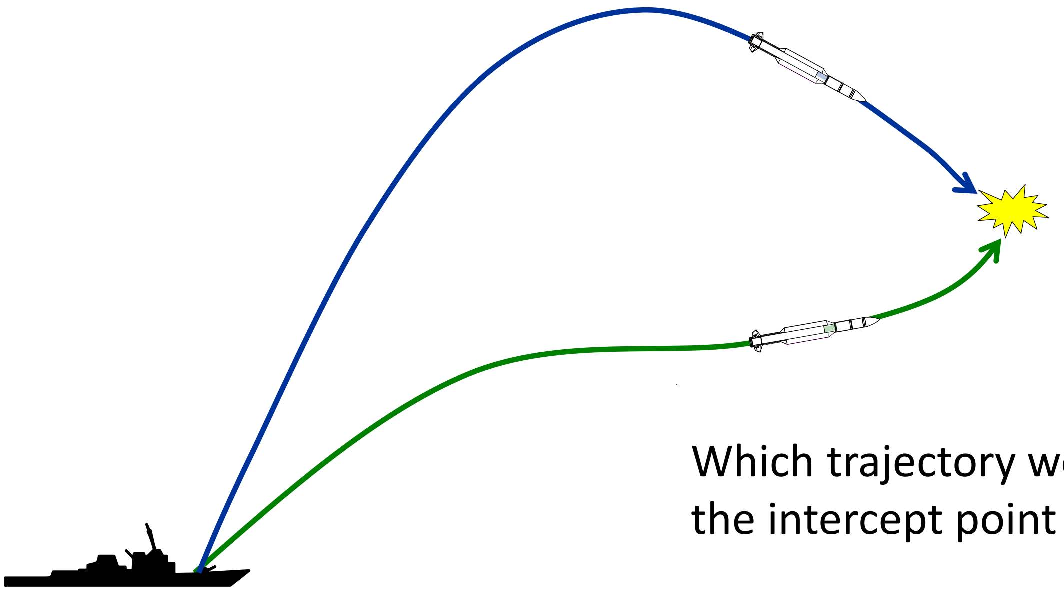
Trajectories

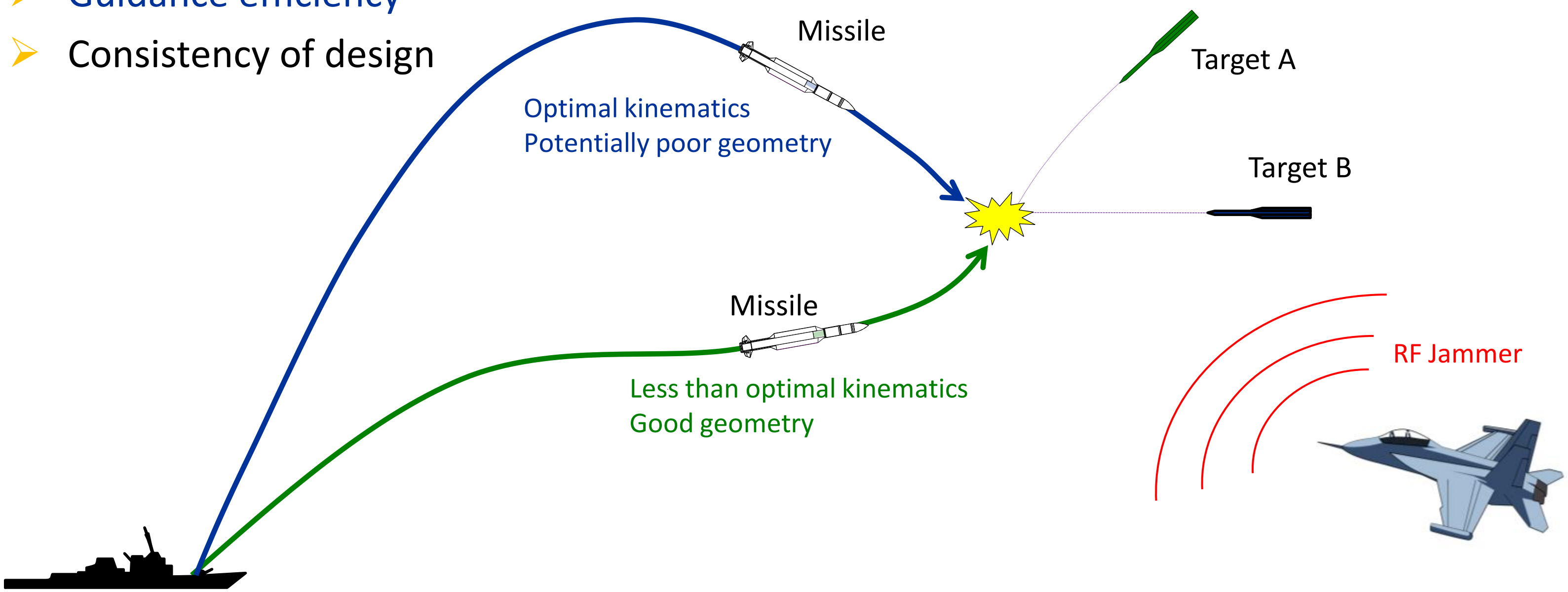
- A trajectory describes the flight path of the interceptor as it moves from point A to point B
- The path in which an interceptor gets from point A to point B can be critical to the success of an engagement



Which trajectory would be preferred to the intercept point shown?

What is a Preferred Trajectory?

- The preferred trajectory can only be determined after considering
  - Threat speed and orientation
  - RF environment
  - Guidance efficiency
  - Consistency of design



The Preferred Solution is Often an Imperfect Solution

Trajectory Design Considerations

- A good trajectory design satisfies many diverse requirements while attempting to optimize multiple performance metrics
  - The resultant balancing act is the heart of trajectory design
- Key parameters of a trajectory which are to be considered
  - Intercept range
  - Intercept velocity
  - Time of flight
  - Intercept geometry

Intercept Range

Trajectory Design Considerations

- Extend intercept range to increase depth of fire (DOF)
  - Creating more opportunities for launches against a given target
- Increase the area which can be covered by an interceptor
  - Defend a larger area
  - Defend more assets
- Push enemy forces further away from the launch platform
  - Enemy surveillance aircraft
  - Enemy electronic attack aircraft
  - Enemy launching platforms

Design trajectories that maximize the range of the interceptor

Intercept Speed

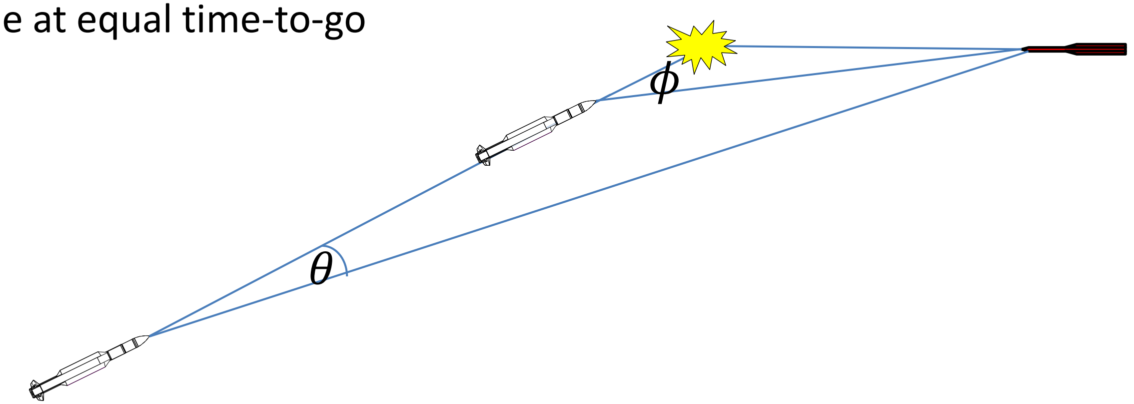
Trajectory Design Considerations

- Increase maneuver capability during terminal guidance
  - Interceptor maneuverability is a function of Mach

$$n_z = \frac{N}{W} = \left(\frac{C_N}{\alpha}\right)\frac{\alpha\, Q\, S_{Ref}}{W} = \left(\frac{(0.7)(P)\left(C_{N_\alpha}\right)(\alpha)\, S_{Ref}}{W}\right) M^2$$

- Improve performance against outbound targets
- Reduce the interceptor to target line of sight (look angle)
  - Smaller look angle at equal time-to-go

$$\theta_{tgo} < \phi_{tgo}$$



Design trajectories that maximize the speed of the interceptor at intercept

Intercept Time

Trajectory Design Considerations

- Increase system reaction time
  - Hit the target before it hits you
  - Increase depth of fire
- Reduce system congestion
  - Reduced radar resources
  - Reduce illumination resources in home-all-the-way applications
  - Less time in the air means more missiles per hour can be fired and supported

Design Trajectories that Minimize Interceptor Time of Flight

Intercept Geometry

Trajectory Design Considerations

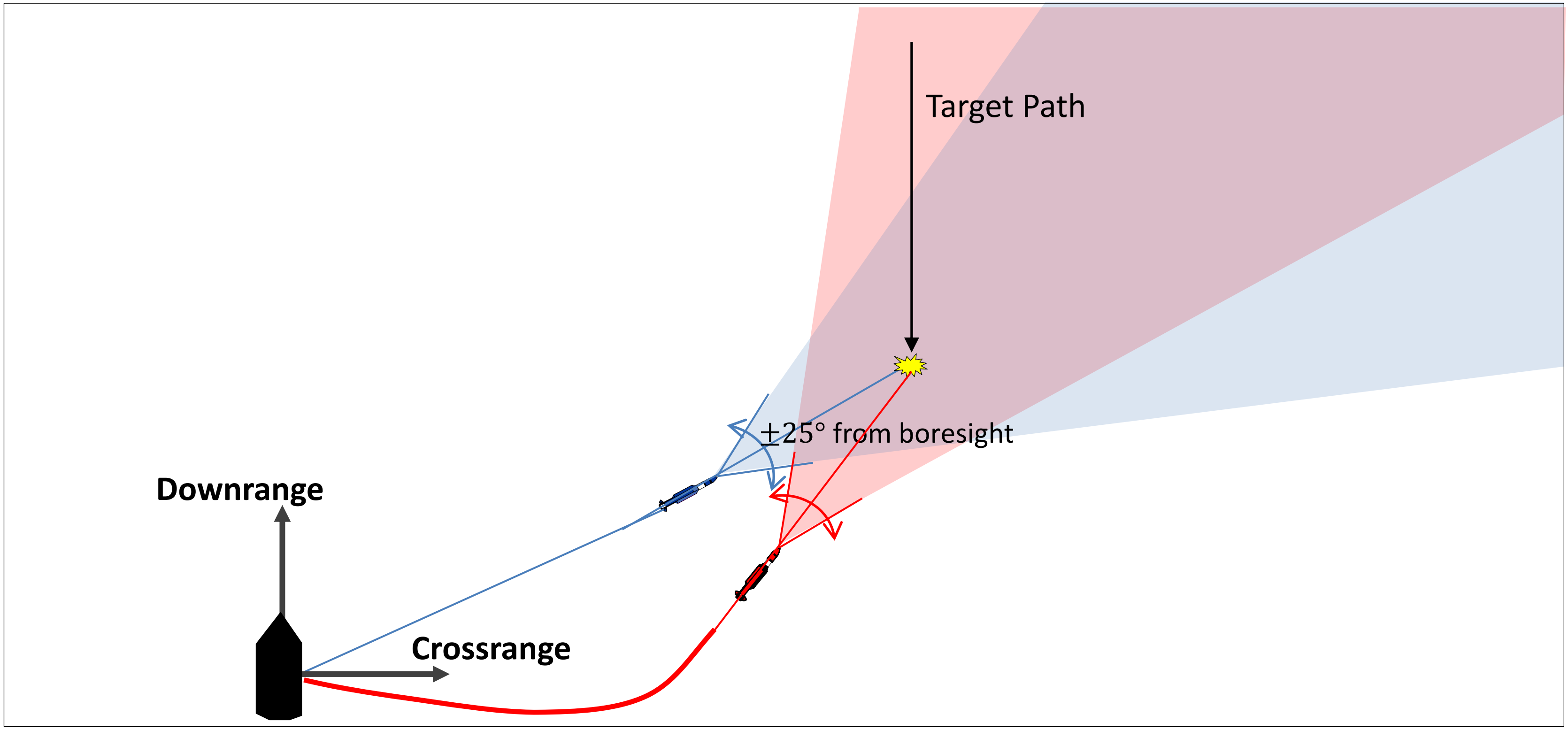
- Increase capability against crossing targets
  - Reduce look angle such that it is within seeker limits
- Increase probability of acquiring target at low altitude
  - Modify interceptor approach angle to reduce multipath effects
- Eliminate large maneuvers in terminal guidance
  - Small heading error at handover (target acquisition by interceptor seeker)
- Improve endgame performance
  - Increase fuze effectiveness by considering terminal crossing angle

Design Trajectories that Balance Many Scenario Specific Requirements

Crossing Target Capability

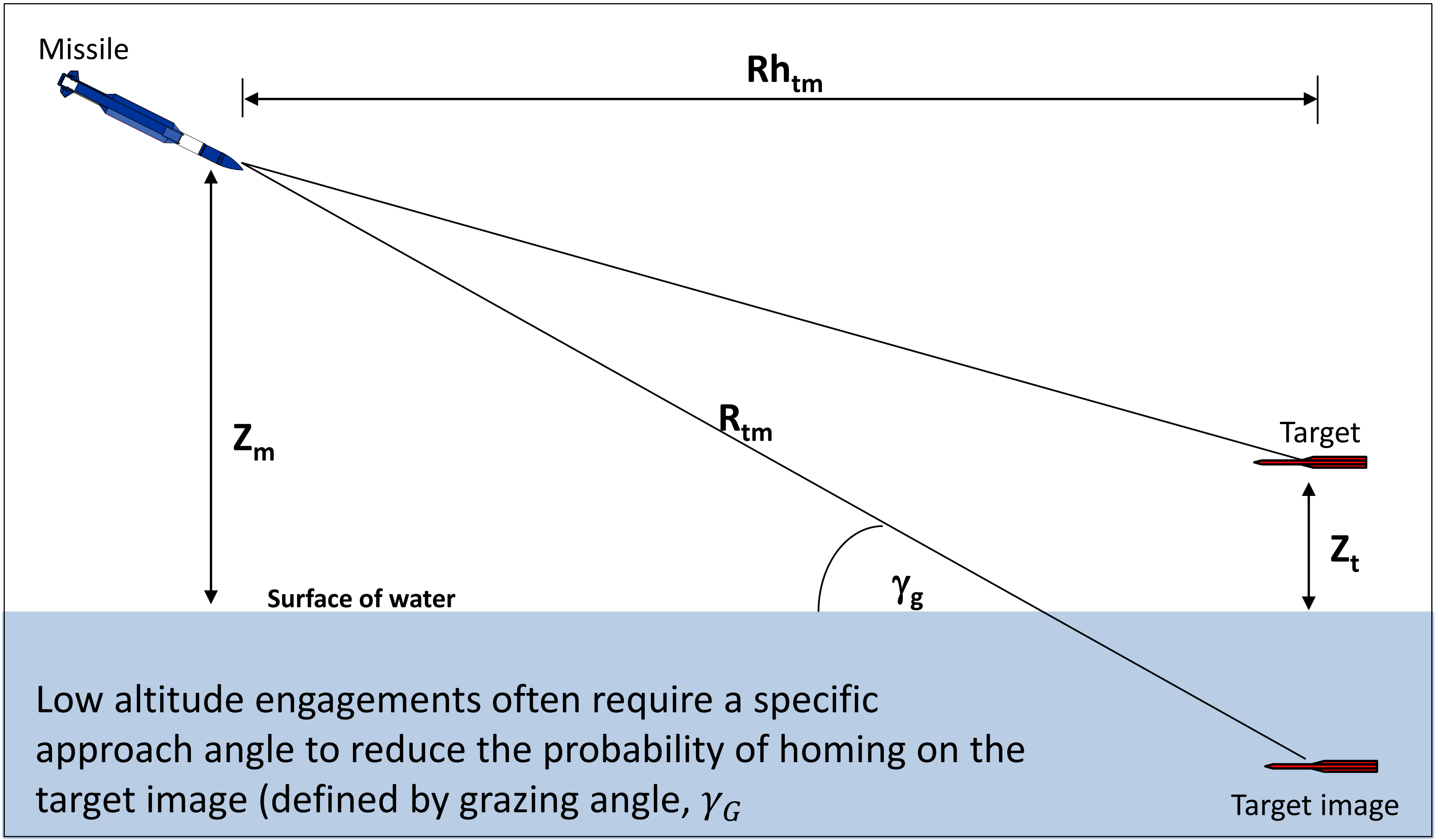
Intercept Geometry

- The missile speed, target speed, and seeker gimbal limit define the crossing capability of a missile vs a given target
- Crossing capability can be expanded by introducing horizontal shaping to keep the target within the seeker gimbal limit during the period in which the missile searches for the target



Low Altitude Approach Angle

Intercept Geometry



Low altitude engagements often require a specific approach angle to reduce the probability of homing on the target image (defined by grazing angle, γ_G

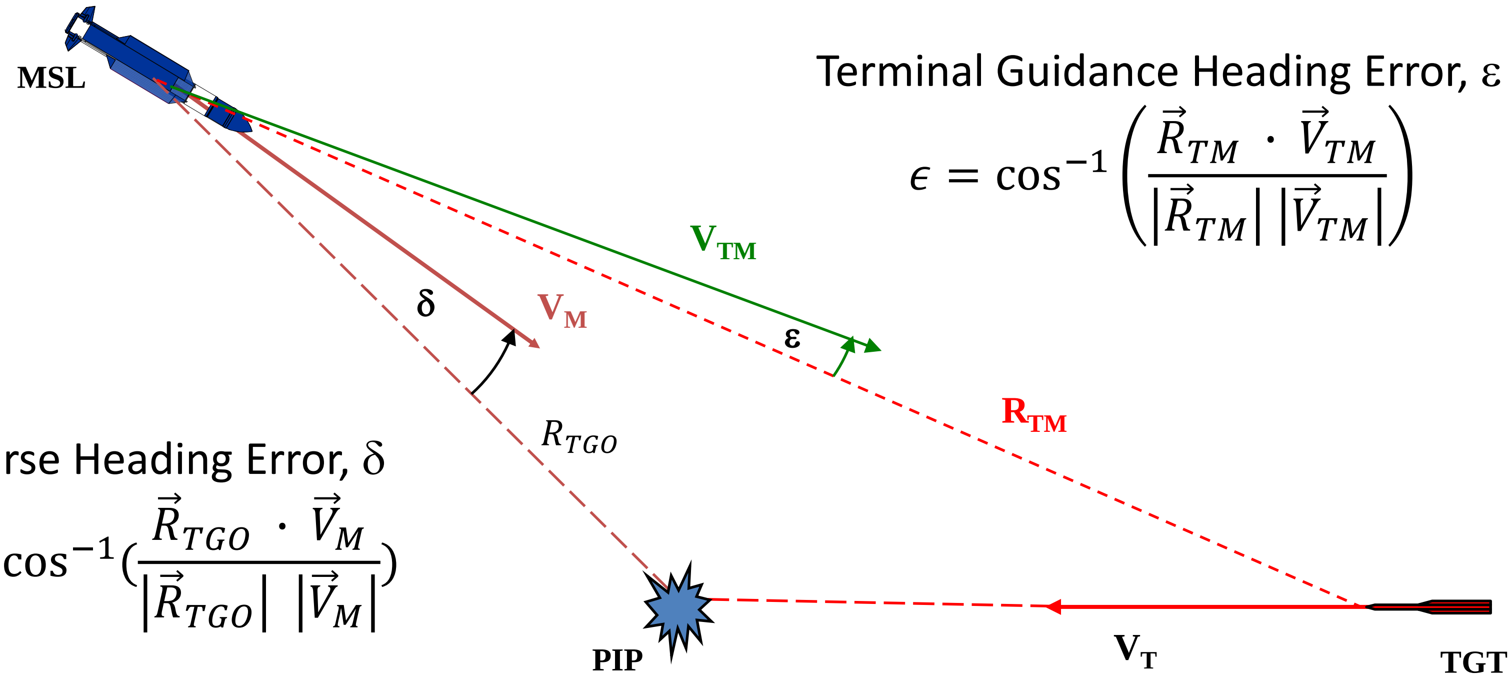
Terminal Heading Error

Intercept Geometry

- The missile trajectory should allow for a small terminal heading error, ϵ
- Since $\delta \neq \epsilon$, one cannot assume a small midcourse heading error (heading error to an intercept point) will guarantee a small terminal guidance heading error

Terminal Guidance Heading Error, ε

$$\epsilon = \cos^{-1}\left(\frac{\vec{R}_{TM} \cdot \vec{V}_{TM}}{|\vec{R}_{TM}|\,|\vec{V}_{TM}|}\right)$$



Midcourse Heading Error, δ

$$\delta = \cos^{-1}\left(\frac{\vec{R}_{TGO} \cdot \vec{V}_{M}}{|\vec{R}_{TGO}|\,|\vec{V}_{M}|}\right)$$

Trajectory Design Factors

- Factors that Influence Trajectory Design
  - Drag Characteristics
  - Propulsion Profile
  - Missile Kinematics
  - Missile/Mission Constraints
- These factors define
  - The physical characteristics of the missile
  - Limitations of the missile to which the trajectory must adhere

Optimal Trajectories are with Respect to a Specific Missile Type

Drag Characteristics

Influences on Trajectory Design

- From our aerodynamics lecture, we learned drag can be described in terms of force coefficients (C_A, C_N), and the interceptor kinematics at a given time

$$C_D \cong C_A + C_N \, \alpha$$

$$Drag \cong 0.7 \, P \, M^2 \, C_A + \frac{n_z^2 W^2}{\left(0.7 \, P \, M^2 \, S_{Ref}\right)^2 \, C_{N_\alpha}}$$

- If one was to minimize the total drag on the interceptor over the trajectory without constraints (or restrictions), the interceptor's final speed would be maximized
  - It should be noted that Mach and altitude are the dominant contributors to the computation of drag
  - $P, C_A, and\ C_{N_\alpha}$ are all functions of Mach and/or altitude
- An optimal trajectory if often defined as a trajectory that maximizes the final speed
  - This is the same as minimizing the interceptor drag
  - This is often simplified to develop tractable guidance solutions

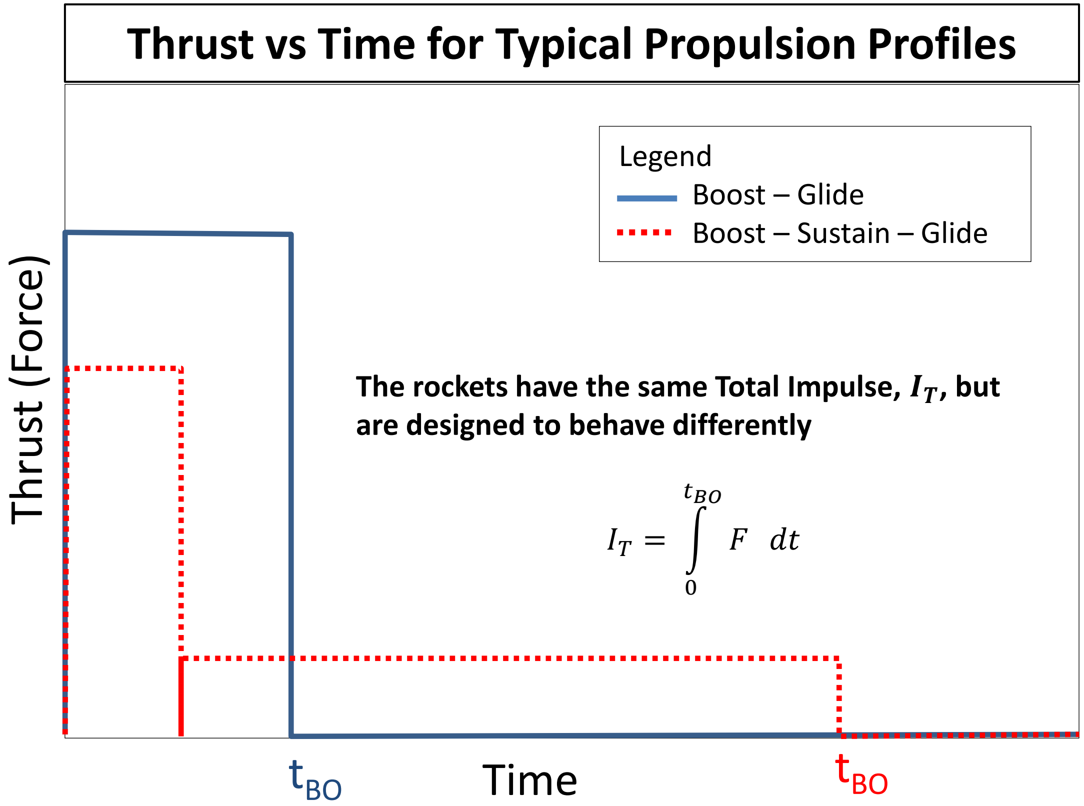
Propulsion Profile

Influences on Trajectory Design

- Propulsion profiles describe the basic characteristics of a rocket. Terms used to describe the rocket propulsion are
  - Boost — A high thrust (typically short-duration) motor burn
  - Sustain — A low thrust (typically long-duration) motor burn
  - Glide — Rocket motor is off
- Some basic rocket profiles include
  - Boost – Glide
    - Large velocity variation over the flight
    - Efficient use of rocket motor
  - Boost –Sustain – Glide
    - Moderate velocity variation over flight
  - Boost – Thrust Controlled Sustain
    - Control of interceptor velocity
    - Used by jets (ramjet, turbojet)

Propulsion Profiles

- The illustration below shows the most common thrust vs time for the most common of the propulsion profiles



Kinematics

Influences on Trajectory Design

- Turning Inertia
  - Maneuver Drag
  - There is always a penalty for generating lift
- Gravity
  - The interceptor must always fight gravity
  - Requires a normal acceleration ($\mathrm{N_{Gravity}} = \mathrm{K_{Gain}} \cos(\gamma)\, G$) to negate the effect of gravity throughout flight
- Maneuver Capability (G-Limits)
  - Required for target maneuvers
  - Required to overcome noise in the guidance loop
  - Structural limitations (max maneuver limit)

High Lift Effectiveness (C_N/α) and High G-Limits Provides Good Guidance Kinematics

Trajectory Shape Options

- The options for a trajectory shape can fall into 4 categories
  - Linear (short range)
  - Constant Mach (ramjet)
  - Ballistic
  - Optimum lift to drag
- Most systems rely upon a combination of the four categories of trajectory shapes

Trajectory Shape Analysis

- Even the simple concept of maximizing interceptor speed can result in a daunting mathematical problem
  - Rocket motor phases
  - Complexities of drag computations
  - Atmospheric considerations (altitude dependent quantities)
- For simplicity, a brute force method is often the preferred method for analyzing interceptor trajectory performance
  - Simulations are used to perform a parametric analysis of various trajectories, using the different guidance parameters, to the same intercept point
  - Key metrics for each flight are analyzed
    - Desired guidance parameters are determined or,
    - Modifications are made to the guidance policy and the study must be repeated

Historical Perspective

Trajectory Shape Analysis Concepts

- Trajectory shaping analysis was done using simplification and approximation techniques
- This provided some very practical (and clever) insight into the development of trajectory shapes
  - Qualitative information is given but definitive performance values could not be obtained
  - By constraining the problem to a subset of conditions, the qualitative results would be used for the practical trajectory synthesis
- The law of energy conservation was the basis for most of this work

$$\mathrm{E_I} = E_D + E_R$$

  - Where
    - E_I = Energy Input into the System (rocket thrust)
    - E_D = Energy Dissipated (drag)
    - E_R = Energy Remaining (kinematic and potential energy)

Energy Conservation

Trajectory Shape Analysis Concepts

- Considering the Law of Energy Conservation in the missile*, we have

$$\int Thust \;\; ds = \int Drag \;\; ds + \int \frac{W_G}{G} V \, dV + \int W_G \;\; dh + \int \frac{W_M}{G} V \, dV \int W_M \;\; dh$$

  - Where
    - s is the incremental path length of the trajectory
    - V is the interceptor velocity
    - h is the interceptor altitude
    - W_G is the weight of the rocket grain
    - W_M (constant) is the weight of the interceptor not including the W_G
  - And the contributors are color coded as such
    - Rocket (slight dependence on altitude)
    - Drag (dependent on altitude and Mach)
    - Grain (dependent on altitude and velocity)
    - Remaining energy (dependent on altitude and velocity)

* From reference 1

Energy Conservation

Trajectory Shape Analysis Fundamentals

- Certain fundamental truths are critical for energy conservation
  - All paths to a given altitude which results in a given velocity have the same remaining energy
  - The criterion for comparing the merit of different trajectories to a given point is the velocity of the interceptor at that point
- The optimum trajectory maximizes the interceptor velocity at that point
  - For a given point, potential energy is constant
  - Maximizing the velocity maximizes the kinetic energy as well as the remaining energy
- If we only allow trajectory variations after rocket burnout our energy equation is simplified

$$E_{Thrust} = E_{preburnout\ drag} + \int_{S_{burnout}}^{S_{final}} Drag\, ds + E_{Grain} + \frac{1}{2}\frac{W_M}{G}V_{final}^2 + W_M\left(h_{final} - h_{inital}\right)$$

Optimization criterion

- Only the drag integral and the remaining kinetic energy are variables
- The maximum final velocity is achieved by minimizing the drag energy integral

Energy Conservation

Trajectory Shape Analysis Fundamentals

- Evaluation of the drag integral gives the designer insight into how the missile is to behave
  - Desired cruising altitude
  - Optimal turn
- The drag integral can be used to find the cruise altitude at which the drag is minimized – i.e. the altitude at which the interceptor motion is most efficient
  - Long range, high-altitude missiles should be efficient at high altitude
  - Make sure your missile is physically well suited for its mission
- Our work can be used to find the optimal turn for the missile (lowest induced drag)
  - Allows for optimal course corrections
  - Useful for a missile that works with waypoints

We'll Investigate Optimal Cruising Altitudes and Optimal Turns

Cruise Altitude (1 of 3)

Trajectory Shape Analysis Fundamentals

- The drag energy integral can be used to provide an approximate optimum trajectory solution

$$\int Drag\ ds = \int \left(Q\ S_{ref}\ C_A \cos(\alpha) + n_z\ W_M \sin(\alpha)\right) ds$$

- By definition:

$$\alpha = \frac{n_z\ W_M}{Q\ S_{Ref}\ C_{N_\alpha}}$$

- Using small angle approximations and the definition of α

$$\int Drag\ ds = \int \left(Q\ S_{Ref}\ C_A + \frac{n_z^2\ W_M^2}{Q\ S_{Ref}\ C_{N_\alpha}}\right) ds$$

- We can treat C_A and C_{N_α} as (approximate) constants, and for a constant altitude, dynamic pressure is not a function of trajectory

Cruise Altitude (2 of 3)

Trajectory Shape Analysis Fundamentals

- We must minimize the integral with respect to Q and set it equal to zero

$$\frac{\partial}{\partial Q}\int Drag\ ds = \frac{\partial}{\partial Q}\int \left(Q\ S_{Ref}\ C_A + \frac{n_z^2\ W_M^2}{Q\ S_{Ref}\ C_{N_\alpha}}\right) ds = 0$$

$$\int \frac{\partial}{\partial Q}\left(Q\ S_{Ref}\ C_A + \frac{n_z^2\ W_M^2}{Q\ S_{Ref}\ C_{N_\alpha}}\right) ds = 0$$

$$\int \left(S_{Ref}\ C_A - \frac{n_z^2\ W_M^2}{Q^2\ S_{Ref}\ C_{N_\alpha}}\right) ds = 0$$

- The function is minimized when the term inside the parenthesis vanishes

$$Q = \frac{n_z\, W_M}{S_{Ref}}\sqrt{C_A\ C_{N_\alpha}} \xrightarrow{\text{yields}} Q_{opt} = \frac{W_M}{S_{Ref}}\sqrt{C_A\ C_{N_\alpha}}$$

  - Since we desire to maintain a constant altitude,
    - $n_z = 1$
    - Remember n_z represents acceleration in units of "G"

Cruise Altitude (3 of 3)

Trajectory Shape Analysis Fundamentals

- Finally, we determine an approximation for the optimal cruise altitude

$$Q \cong 1481 \frac{P_h}{P_{sl}} M^2 \approx 1481\, M^2 \exp\left(\frac{h}{23{,}000}\right)$$

  - Where
    - M is Mach
    - $\frac{P_h}{P_{sl}}$ is the ratio of atmospheric pressure at altitude to pressure at sea level
    - $Q = \frac{\gamma}{2} P\, M^2 \cong 1481 \frac{P_h}{P_{sl}} M^2$ is a common approximation for dynamic pressure
      - γ is the specific heat ratio of air
      - P is ambient pressure (lbs/ft^2)

- Setting the above equal definition of $\mathrm{Q} = Q_{opt}$ and solve for the optimal cruise altitude

$$h_{opt} \approx 23{,}000 \ln\left(\frac{W_M}{S_{Ref}\, M^2 \sqrt{C_A\, C_{N_\alpha}}}\right)$$

The Optimal Cruise Altitude is Only a Function of Mach

Optimal Turn (1 of 2)

Trajectory Shape Analysis Fundamentals

- Starting with the drag integral with which we've assumed small angle approximations

$$\int Drag\ ds = \int \left(Q\ S_{Ref}\ C_A + \frac{n_z^2\ W_M^2}{Q\ S_{Ref}\ C_{N_\alpha}} \right) ds$$

- In order to develop the optimal level of maneuver, the path length ds needs to be expanded upon

$$ds\ = R\ d\gamma = \frac{V^2}{n_z\ G}\ d\gamma$$

  - Where
    - γ is the interceptor's heading
    - R is the radius of the turn

- Substituting the expression for ds into the drag equation yields

$$\int Drag\ ds = \int_{\gamma_0}^{\gamma_f} \left(\frac{Q\ S_{Ref}\ C_A}{n_z} + \frac{n_z\ W_M^2}{Q\ S_{Ref}\ C_{N_\alpha}} \right) \frac{V^2}{G}\ d\gamma$$

Optimal Turn (2 of 2)

Trajectory Shape Analysis Fundamentals

- To determine the optimal acceleration, we take the partial of the previous equation with respect to n_z

$$\frac{\partial}{\partial n_z}\left[\int Drag\ ds\right] = \int_{\gamma_0}^{\gamma_f}\left(\frac{W_M^2}{Q\ S_{Ref}\ C_{N\alpha}} - \frac{Q\ S_{Ref}\ C_A}{n_z^2}\right)\frac{V^2}{G} = 0$$

- The term in parenthesis vanishes when

$$n_{z_{opt}} = \frac{Q\ S_{Ref}}{W_M}\sqrt{C_A\ C_{N\alpha}}$$

⟺ Optimum maneuver level for a turn

- Turn radius for the optimal turn can easily be found

$$R_{opt} = \frac{V^2}{n_z\ G}$$

The Optimal Turn is a Function of Mach and Altitude

Generating Optimal Trajectories

- Gaining an understanding of the interceptor's preferred regions of operation is important for trajectories with long cruise phases
  - Optimal cruise altitude
  - Optimal turns
- A more accurate solution must be considered when synthesizing a trajectory which is meant to be folded into a robust weapon system
  - High depth of fire
  - Area defense considerations
  - Optimal terminal speed / good intercept geometry
  - Etc.
- The complexities of the robust solution make this problem mathematically challenging
  - A trajectory analysis process must be invoked to develop trajectories that work well across the battlespace

Trajectory Analysis Process

Analysis Process (Mid 20th Century)

1. Select desired trajectory shape
2. Select form of guidance law using simplified system equations and intuition
3. Tune guidance law to obtain desired shape
4. Expand number of intercept points
5. Insert noise, tolerances into analysis
6. Evaluate special threats (if any)
7. Modify guidance law (if necessary)
8. Repeat steps 3-8 for each intercept point until each intercept point has satisfactory performance and transitions between intercept points are acceptable

Analysis Process (Late 20th and 21st Century)

1. Select desired trajectory shape
2. Select form of guidance law using optimal control theory
3. Conduct study varying all guidance parameters parametrically
4. Data mine for the best subset of trajectories given specified criteria
5. Find the guidance parameters for each intercept point which allows for an acceptable transition between neighboring intercept points

Inter-Intercept Point Trajectory Analysis

- As guidance parameters change from intercept point to intercept point, care must be given to ensure robust system performance is guaranteed temporally and spatially
- Temporally
  - Change in time of flight as a function of range is gradual to avoid “holes” in scheduling algorithms
  - Time of flight to each intercept point increases monotonically as a function of range
  - Contour plots are a fantastic way to evaluate this criteria, but requires artistic evaluation
    - More often then not, a human must evaluated “goodness of fit” of the trajectory solutions across the battlespace
- Spatially
  - Trajectories should not overlap in the horizontal or vertical planes to reduced risk of fratricide
  - Imposes a constraint on start and end point parameter selection for trajectory shaping

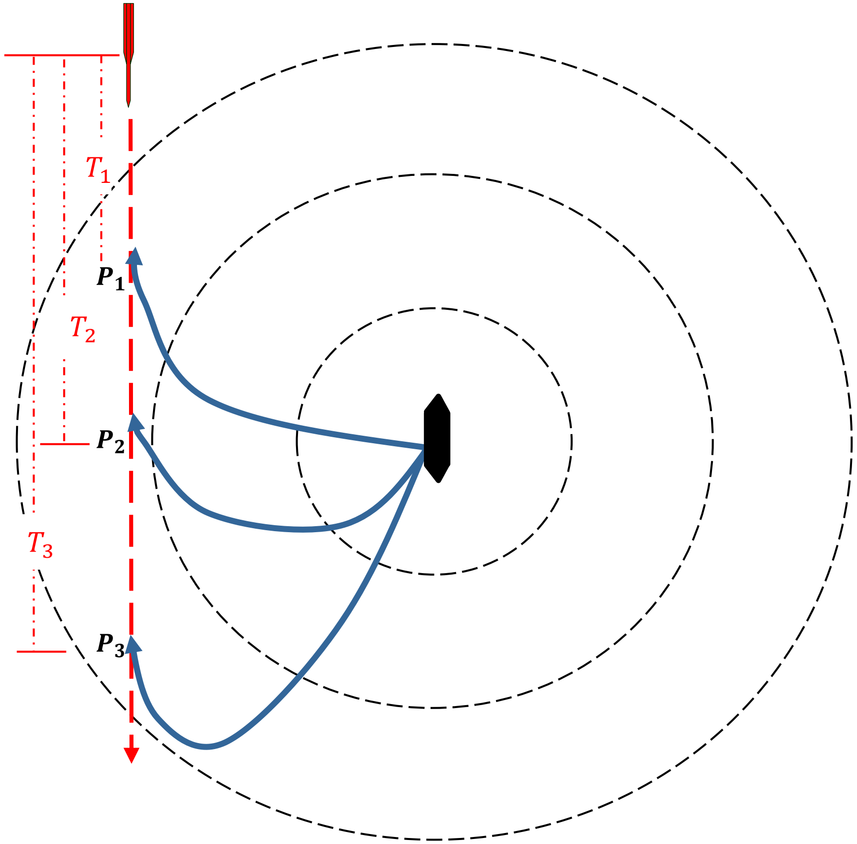
Time of Flight Constraints

Inter-Intercept Point Trajectory Analysis

- Robust trajectory shaping designs consider the time of flight of each intercept point in relation to adjacent points
- Consider the time of flight (TOF) for point P_1, P_2, and P_3
- The time it takes for the target to arrive at those points are noted as T_1, T_2, and T_3
- In order for the scheduler to have a valid firing solution for all points along the target's path, the following must be true

$$TOF_{R_a} > TOF_{R_b} \text{ if } R_a > R_b$$

  - TOF_{R_x} is the time of flight at range R_x from the firing platform
- The use of shaping in one area of the battlespace may force shaping in other areas of the battlespace



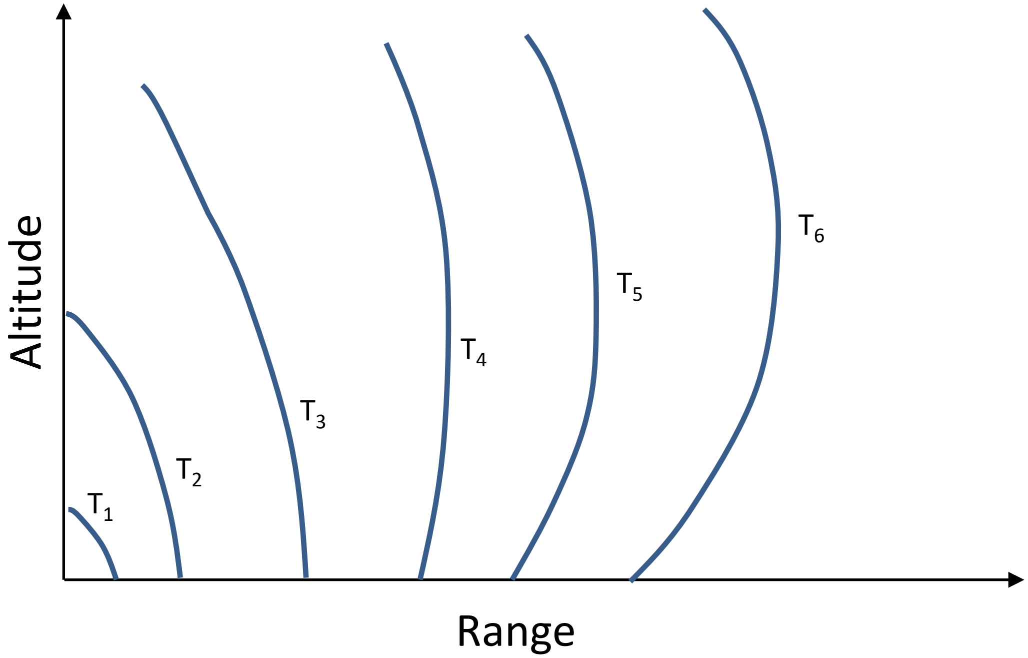
Standard Measures of Trajectories

- One uses specific characteristics of trajectories over an entire battlespace to verify a complete system design has been achieved
- Characteristics of interest
  - Time line contours
  - Maneuver contours / Mach line contours
  - Trajectory shapes
  - Intercept boundaries / Engagement boundaries

Timeline Contours

Standard Measures of Trajectories

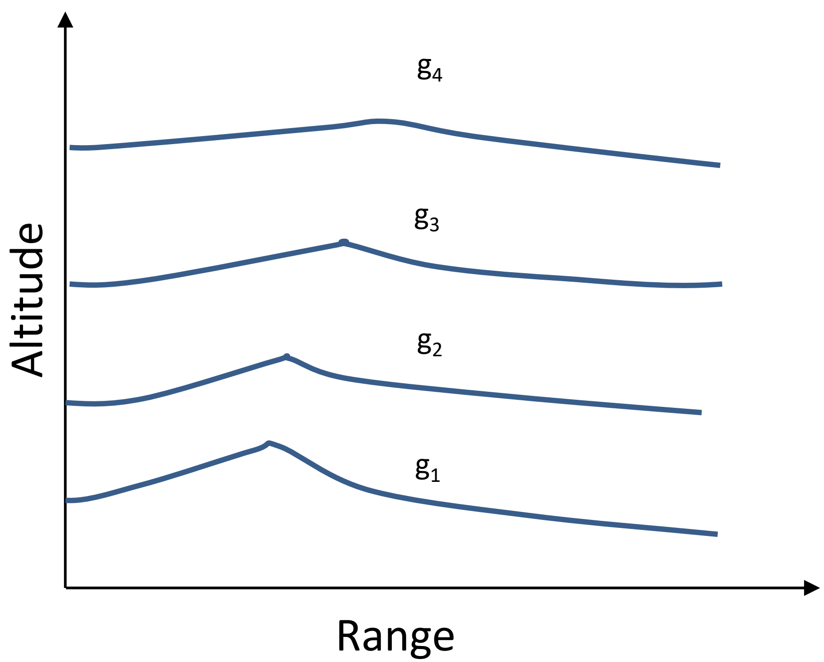
- The basic requirement for the timeline contour is
  - $T_1 < T_2 < \ldots\ldots\ T_N$
  - Some consideration must be given to the spacing of the contour lines such that “steps” or “jumps” are not present

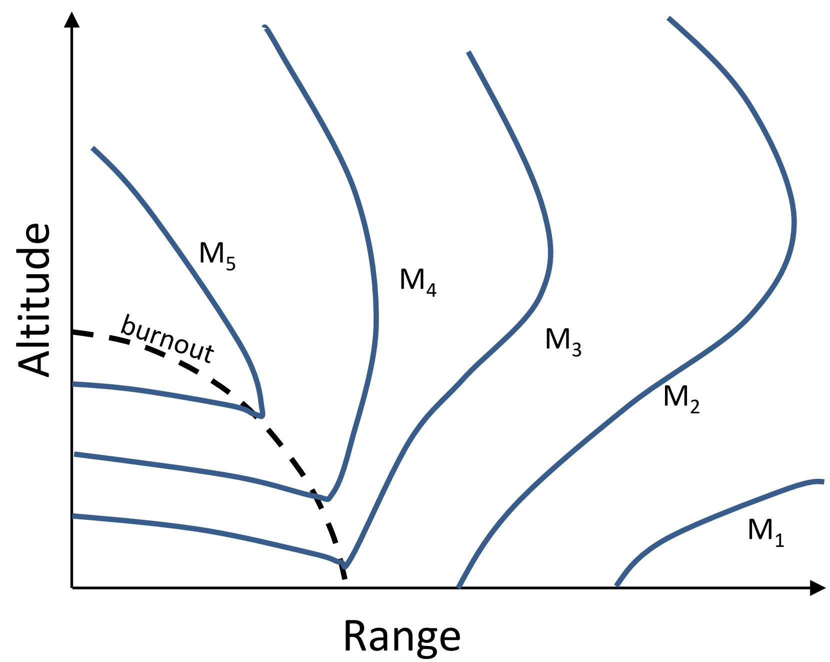


Maneuver / Mach Contours

Standard Measures of Trajectories

- The maneuver (or G) contour must maintain a minimum maneuver potential throughout intercept
  - $(g_1 > g_2 > \dots g_N) > g_{min}$
- The Mach contour helps satisfy some basic aerodynamic stability requirements throughout flight
  - $M_x > M_{min}$

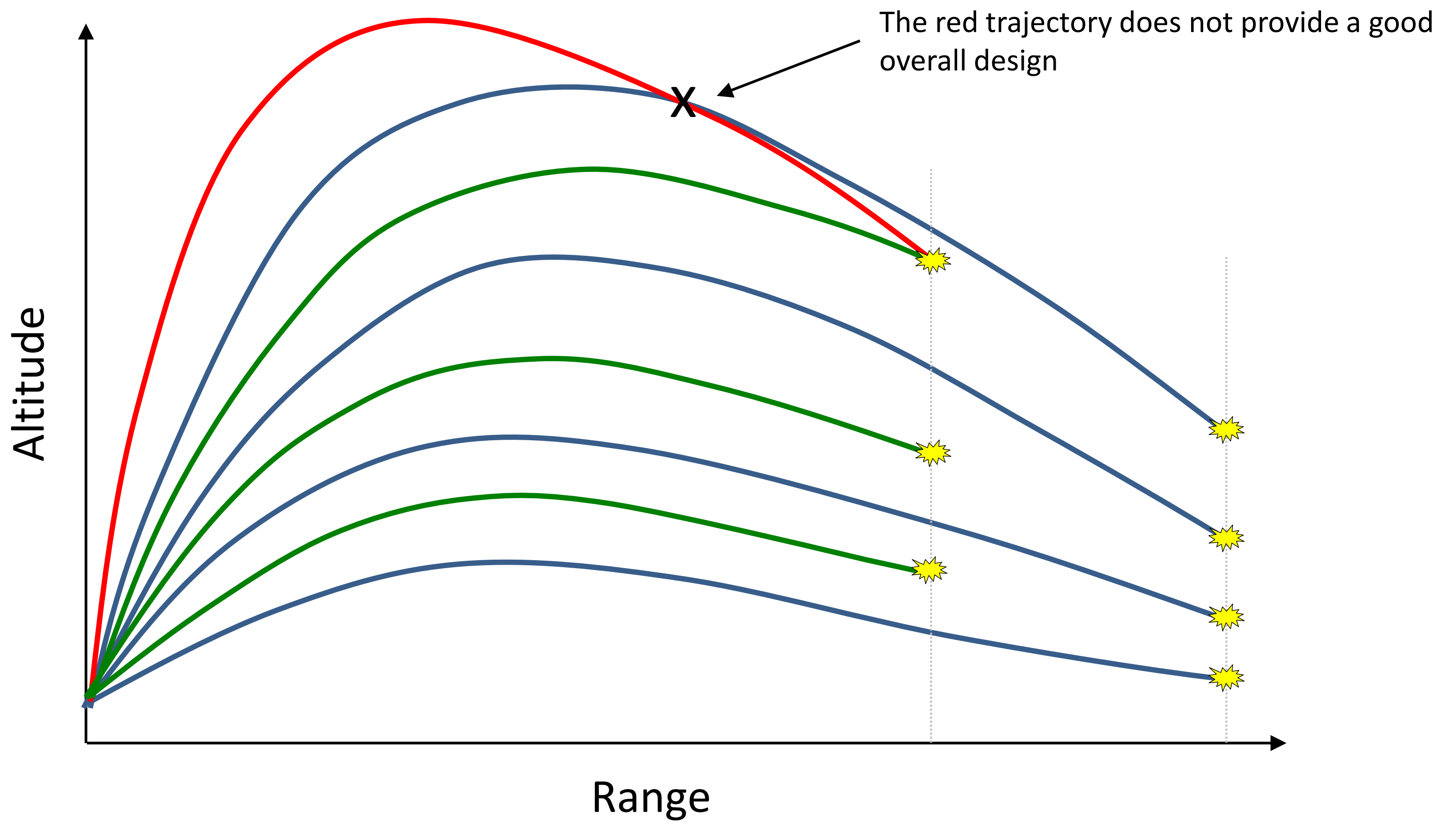




Trajectory

Standard Measures of Trajectories

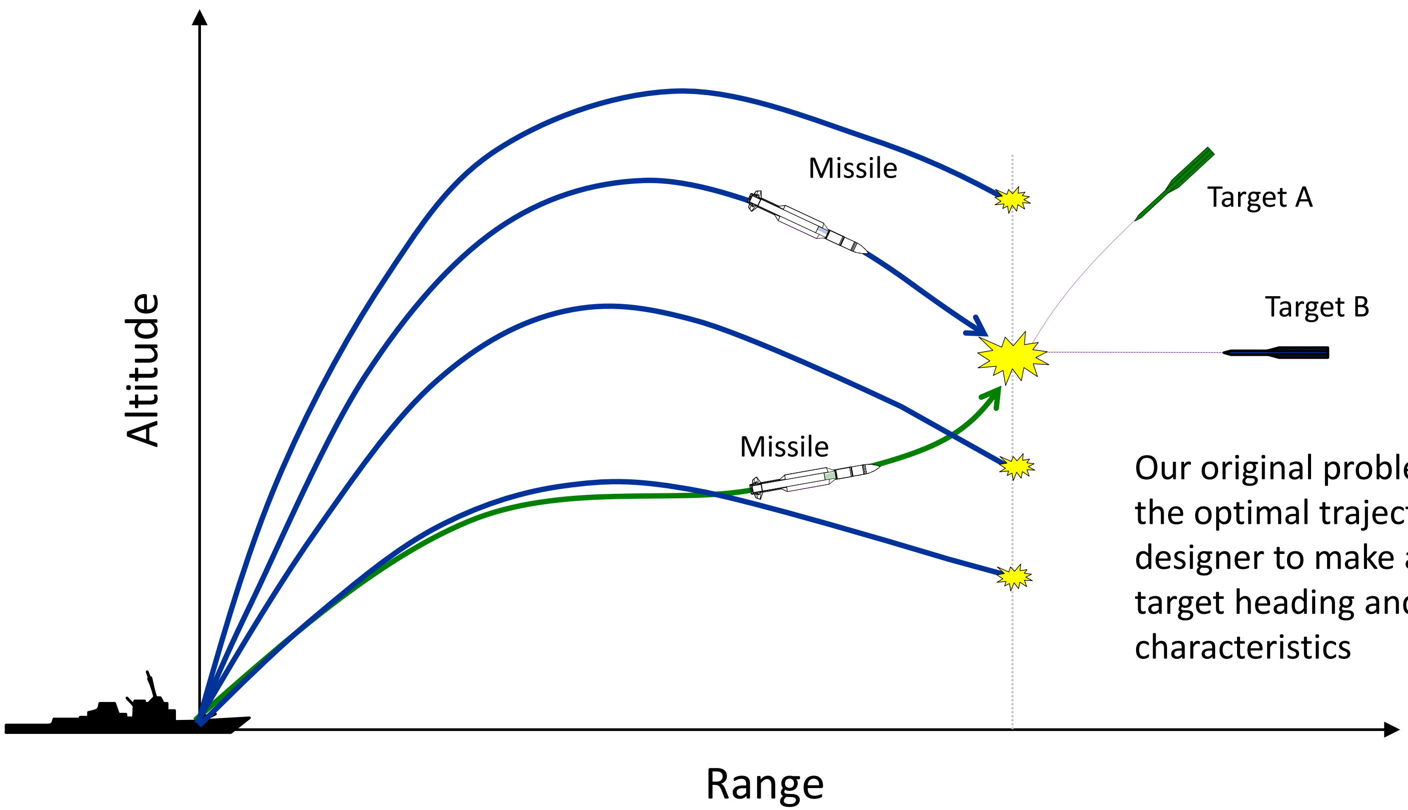
- Trajectories should be "well behaved" – meaning the trajectory lines should never touch
  - This reduces the probability of fratricide
  - This increases the probability of monotonic time of flight across the battlespace



Trajectory Implications

Standard Measures of Trajectories

- The need to ensure trajectories don't overlap results in the guidance policy being consistent across the battlespace OR a multiple guidance policies are required and a guidance policy selection algorithm must be incorporated



Our original problem of selecting the optimal trajectory forces the designer to make assumptions on target heading and/or jamming characteristics

References

- 1. Lange, Steve. Missile Trajectory Design. Missile System Engineering Fundamentals, Lockheed Martin Course, ~1984