

VLSI Homework 1 - Aidan Sharpe

Problem 1

```
import numpy as np
import matplotlib.pyplot as plt

e_0 = 8.85E-14
k_ox = 3.9
e_ox = e_0 * k_ox

mu = 350          # 350 cm^2 / (V*s)
t_ox = 100E-8    # 100A

C_ox = e_ox / t_ox
W = 1.2
L = 0.6

beta = mu * C_ox * W/L

V_g_range = np.arange(6)
V_t = 0.7
V_dd = np.linspace(0, 5, 50)

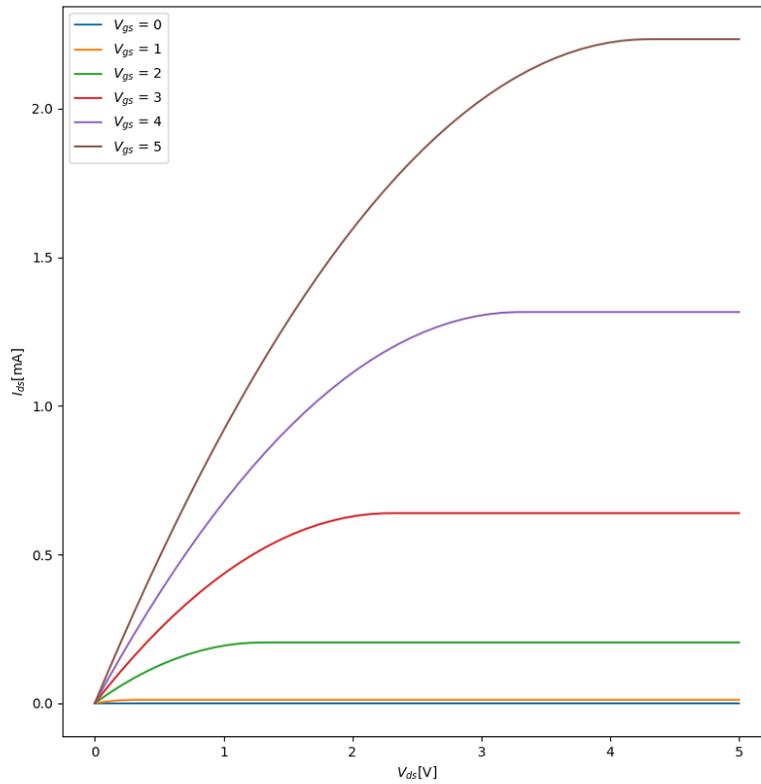
def I_ds(V_gs, V_ds, V_t, beta):
    if V_gs < V_t:
        return np.zeros_like(V_ds)

    V_dsat = V_gs - V_t

    return np.where(V_ds < V_dsat,
                    beta * (V_gs - V_t - V_ds/2) * V_ds,
                    beta * (V_gs - V_t)**2 / 2)

for V_s in (0, 2):
    for V_g in V_g_range:
        V_gs = V_g - V_s
        V_ds = V_dd - V_s
        plt.plot(V_ds,
                 1000*I_ds(V_gs, V_ds, V_t, beta),
                 label="$V_{gs}$ = "+str(V_gs))

plt.xlabel("$V_{ds}$ [V]")
plt.ylabel("$I_{ds}$ [mA]")
plt.legend()
plt.show()
```



Problem 2

Show that a MOSFET with channel length $2L$, when operating in the linear region, has the same current I_{DS1} as two MOSFETs in series each with channel length L . Assume the same gate voltage V_{DD} for all MOSFETs in the design and the same supply voltage V_{DS} for both designs.

Known

The current through a MOSFET in the linear region is:

$$I_{DS} = \beta \left(V_{GS} - V_t - \frac{V_{DS}}{2} \right) V_{DS}$$

The parameter β is defined as:

$$\beta = \mu C_{\text{ox}} \frac{W}{L}$$

Defining Variables

The factor β_1 for the singular MOSFET is $\mu C_{\text{ox}} \frac{W}{2L}$, and the factor β_2 for each of the MOSFETs in series is $\mu C_{\text{ox}} \frac{W}{L}$. Therefore, $\beta_2 = 2\beta_1$.

The current I_{DS1} through the singular MOSFET is:

$$I_{DS1} = \beta_1 \left(V_{DD} - V_t - \frac{V_{DS}}{2} \right) V_{DS}$$

The current I_{DS2} through both MOSFETs in series is both:

$$I_{DS2} = \beta_2 \left(V_{DD} - V_1 - V_t - \frac{V_{DS} - V_1}{2} \right) (V_{DS} - V_1)$$

$$I_{DS2} = \beta_2 \left(V_{DD} - V_t - \frac{V_1}{2} \right) V_1$$

Expanding I_{DS2}

The first equation for I_{DS2} can be rewritten as the difference of two terms:

$$I_{DS2} = \beta_2 \left(V_{DD} - V_t - \frac{V_{DS}}{2} - \frac{V_1}{2} \right) (V_{DS} - V_1)$$

$$I_{DS2} = \beta_2 \left(V_{DD} - V_t - \frac{V_{DS}}{2} - \frac{V_1}{2} \right) V_{DS} - \beta_2 \left(V_{DD} - V_t - \frac{V_{DS}}{2} - \frac{V_1}{2} \right) V_1$$

The term $-\frac{V_1 V_{DS}}{2}$ can be pulled out of the first term, and its complement can be pulled out of the second term. Therefore, I_{DS2} can be simplified to the difference of two terms each in the form of the current through a MOSFET in the linear region.

$$I_{DS2} = \beta_2 \left(V_{DD} - V_t - \frac{V_{DS}}{2} \right) V_{DS} - \beta_2 \left(V_{DD} - V_t - \frac{V_1}{2} \right) V_1$$

Reducing I_{DS2}

Notice that the second term is the inverse of the second equation for I_{DS2} under ‘‘Defining Variables’’. Setting them equal:

$$I_{DS2} = \beta_2 \left(V_{DD} - V_t - \frac{V_{DS}}{2} \right) V_{DS} - \beta_2 \left(V_{DD} - V_t - \frac{V_1}{2} \right) V_1 = \beta_2 \left(V_{DD} - V_t - \frac{V_1}{2} \right) V_1$$

Therefore:

$$\beta_2 \left(V_{DD} - V_t - \frac{V_{DS}}{2} \right) V_{DS} = 2\beta_2 \left(V_{DD} - V_t - \frac{V_1}{2} \right) V_1$$

Substituting β_2 on the left for $2\beta_1$ gives:

$$2\beta_1 \left(V_{DD} - V_t - \frac{V_{DS}}{2} \right) V_{DS} = 2\beta_2 \left(V_{DD} - V_t - \frac{V_1}{2} \right) V_1$$

Therefore:

$$2I_{DS1} = 2I_{DS2}$$

$$\boxed{I_{DS1} = I_{DS2}}$$