Weapon Systems Midterm Equation Sheet

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Aerodynamics

Mach number: $M = \frac{v_{\text{missile}}}{v_{\text{sound}}}$

Dynamic pressure: $Q = \frac{\rho}{2}v^2 \approx 0.7PM^2$, where ρ is ambient density, v is the missile velocity, P is the ambient pressure, and M is the Mach number.

Pressure waves: $\mu = \arcsin\left(\frac{1}{\mathrm{M}}\right)$, the angle of a supersonic shock wave above the direction of motion, where M is the Mach number.

Force coefficients: $C_N = \frac{N}{QS_{\text{Ref}}} \approx C_{N_\alpha}\alpha$, where N is the normal force, S_{Ref} is the maximum cross-sectional area (calculate using diameter of the missile), C_{N_α} is a constant, and α is the angle of attack.

Moment coefficients: $C_m = \frac{m}{QS_{\text{Ref}}L_{\text{Ref}}}$, where m is the pitching moment, and L_{Ref} is the reference length.

Induced drag: $C_{D_I} = C_N\alpha = C_{N_\alpha}\alpha^2 = \frac{1}{C_{N_\alpha}}\left(\frac{n_z W}{QS_{\text{Ref}}}\right)^2$, where n_z is the maneuver acceleration, and W is the missile weight.

Maneuver G's: $n_z = \frac{N}{W} = C_{N_\alpha}\frac{\alpha Q S_{\text{Ref}}}{W}$, where N is the normal force, and W is the missile weight.

Lift: $L = N\cos(\alpha) - A\sin(\alpha) \approx N$, where N is the normal force, and A is axial drag.

Drag: $D = A\cos(\alpha) + N\sin(\alpha) \approx A + N\alpha$, where A is axial drag, and N is normal force.

Static margin: $SM = CG - CP$, where CG is the center of gravity, and CP is the center of pressure. Unstable when $SM > 0$ (CG is aft of CP). As a rule of thumb, $SM \approx -0.5d$, where d is the missile diameter.

Body fineness ratio: $BFR = \frac{l}{d}$, where l is the missile length, and d is the missile diameter. Typically between 5 and 25.

Nose fineness ratio: $NFR = \frac{l}{d}$, where l is the nose length, and d is the maximum nose diameter. Typically between 2 and 4.

Rocket Propulsion

Rocket thrust: $F = \frac{\dot{W}v_e}{g} + (P_e + P_a)A_e = \frac{P_0 A^*}{C^*}v_e + (P_e + P_a)A_e$, where $\dot{W}$ is the propellant weight flow rate, v_e is the exhaust exit velocity, P_e is the exit pressure, P_a is the outside pressure, and A_e is the nozzle exit area.

Mass flow rate: $\dot{m} = \frac{\dot{W}}{g}$

Weight flow rate: $\dot{W} = g\frac{P_0 A^*}{C^*}$, where P_0 is the chamber pressure, A^* is the throat area, and C^* is the characteristic velocity of burned propellants.

Characteristic velocity: $C^* = \frac{223}{K}\sqrt{\frac{T_0}{m}}$, where m is molecular weight, K is a function of the specific heat ratio, and T_0 is the flame temperature.

Specific impulse: $I_{sp} = \frac{F}{\dot{W}}$

Exit velocity: $v_e = gI_{sp}$

Ideal burnout velocity: $V_{BO_I} = gI_{sp}\ln\left(\frac{W_L}{W_{BO}}\right)$, where W_L is the weight of the vehicle at launch, and W_{BO} is the weight of the vehicle at burnout.

Realistic burnout velocity: $V_{BO} = V_{BO_I} - g\sin(\bar{\gamma})T_{BO}$, where $\bar{\gamma}$ is the average flight path angle, and T_{BO} is the time at burnout.

Rocket velocity: $v(t) = v_0 + v_e\ln\left(\frac{m_0}{m(t)}\right) - g\sin(\bar{\gamma})t$, where v_0 is the initial velocity, m_0 is the initial mass, and $m(t)$ is the mass at time t.

Weapon Control Systems

Total engagement time: $TET = \frac{ROF - R_{\min}}{v_t}$, where ROF is the range of open fire, $R_{\min}$ is the range of the final shot, and v_t is the velocity of the target.

Duration of first shot: $TOF_1 = \frac{ROF}{v_m + v_t}$, where v_m is the velocity of the missile, and v_t is the velocity of the target.

Depth of fire: $DOF = \frac{TET - TOF_1}{T_H} + 1$, where T_H is the homing time.

Time between launches: $\Delta T_L = T_H\left(1 + \frac{v_t}{v_m}\right)$, where T_H is homing time, v_t is the velocity of the target, and v_m is the velocity of the missile.

Total launching time: $N_L = N\Delta T_L$

Time to go: $TGO = \frac{|\vec{R}_{TM}|}{\cos(\theta_m)|\bar{v}_m| + \cos(\theta_t)|v_t}$, where R_{TM} is the vector from the missile to the target, θ_m is the angle between $\bar{v_m}$ and $\vec{R}_{TM}$, $\bar{v_m}$ is the average remaining weapon velocity, θ_t is the angle between $\vec{R}_{TM}$ and v_t, and v_t is the target velocity (assumed constant).

Predicted intercept point: $\overrightarrow{PIP} = \vec{R}_T + TGO\vec{v_t}$, where R_T is the current vector from the illuminator to the target.

Power density at the missile seeker: $PDMS = \frac{P_T G_T \sigma_{RCS}}{L_{IL}(4\pi)^2 R_T^2 R_{TM}^2}$, where P_T is the illuminator transmit power, G_T is the antenna gain, σ_{RCS} is the radar cross section of the target, L_{IL} is the total of the transmit losses of the illuminator, R_T is the distance from the RF source to the target, and R_{TM} is the distance from the target to the missile seeker.

Trajectory Design

Midcourse heading error: $\varepsilon = \arccos\left(\frac{\vec{R}_{TGO} \cdot \vec{v}_M}{|\vec{R}_{TGO}||\vec{v}_M|}\right)$

Terminal guidance heading error: $\varepsilon = \arccos\left(\frac{\vec{R}_{TM} \cdot \vec{v}_{TM}}{|\vec{R}_{TM}||\vec{v}_{TM}|}\right)$

Thrust energy optimization: $E_{\text{thrust}} = E_{\text{preburnout drag}} + \int\limits_{s_{\text{burnout}}}^{s_{\text{final}}} \text{Drag} ds + E_{\text{grain}} + \frac{1}{2}\frac{W_M}{g} v_{\text{final}}^2 + W_M(h_{\text{final}} - h_{\text{initial}})$, where s is the incremental path length of the trajectory, v_{final} is the final velocity of the interceptor, h is the interceptor altitude, W_M is the weight of the interceptor without fuel.

Optimal dynamic pressure: $Q_{\text{opt}} = \frac{W_M}{s_{\text{Ref}}}\sqrt{C_A C_{N_\alpha}}$

Cruise altitude: $h_{\text{opt}} \approx 2.3 \times 10^4 \ln\left(\frac{W_M}{s_{\text{Ref}} M^2 \sqrt{C_A C_{N_\alpha}}}\right)$, where W_M is the weight of the empty missile and M is the Mach number.

Optimal turn: $R_{\text{opt}} = \frac{v^2}{n_z g}$