Waveform Synthesis and Spectral Analysis

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I. Background

Waveform synthesis and spectral analysis are fundamental concepts used to generate, analyze, and understand signals. Waveform synthesis is the process of generating specific waveforms that can encode information for transmission in a communication system. This is common in signal encoding, signal modulation, and pulse shaping. On the other hand, spectral analysis is when a signal is studied in the frequency domain to understand its frequency components. This is most commonly applied to frequency component identification, noise and interference detection, channel capacity and bandwidth allocation, and modulation analysis.

II. Introduction

For the first portion of the lab, we are tasked with synthesizing a waveform using a specified signal to noise ratio (SNR). This included synthesizing the signal, plotting it on a graph, corrupting it with noise to get an SNR of 10, and synthesizing one cycle of the new noisy waveform. This process was repeated for different SNRs. After this, the second portion of the lab required comparing a continuous Fourier transform (CFT) with a discrete Fourier transform (DFT). The CFT was found analytically and plotted, then compared to the plot of the DFT using MATLAB and discussed. The third portion of this lab focused on the spectral analysis of AM and FM signals. This was done by first synthesizing an AM signal, finding the spectral components using MATLAB, adding noise to the RF signal, and repeating using various different values. After this, the same process was completed with an FM signal and comparisons between the two were discussed. The fourth and final part of the lab was spectral analysis of a piece of music which was accomplished by plotting the frequency spectrum of the song and observing its characteristics. Overall, this lab was focused around waveform synthesis and spectral analysis.

III. Generating Arbitrary Waveforms with Specified SNRs

We were provided with sample code to assist our programming. The code produced a one second long A♯ signal, which is a sinusoidal signal with frequency $f_{\mathrm{A}\sharp} = 433.16$[Hz]. This signal was sampled at 8 kHz, and altered to plot the waveform as shown in figure 1.

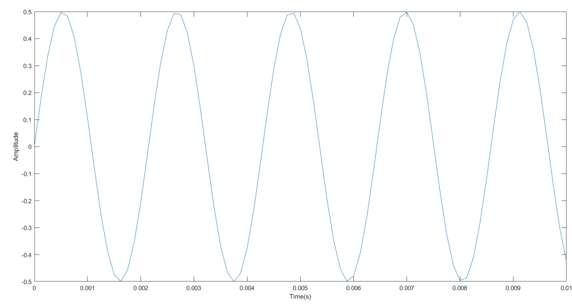

Fig. 1. Time domain representation of a pure A♯ tone with amplitude $\frac{1}{2}$.

In addition, the signal was corrupted with a Gaussian noise source to get a SNR of 10 dB. This waveform sounded staticky and weak compared to the first signal. We then repeated the process with SNR values of 20 and 30 dB. As seen in figure 2, increasing the SNR increased the clarity of the signal.

Fig. 2. Time domain representation of the same signal with an SNR of 10db

IV. Differences between the Continuous and Discrete Fourier Transform

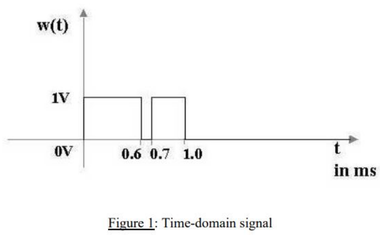

Fig. 3. Time domain plot of $w(t)$

We are given continuous time signal, $w(t) = u(t) - u(t - 0.6) + u(t + 0.7) - u(t - 1)$, seen in figure 3. Since it is a bounded, finite support aperiodic signal, it is absolutely integrable. Therefore, its Continuous Fourier transform (CFT) exists. The CFT of a signal is given by

$$\mathcal{F}\{w(t)\}(\omega) = \int_{-\infty}^{\infty} w(t) e^{-j\omega t} dt \tag{1}$$

where ω is angular frequency. Evaluating the integral gives

$$\mathcal{F}\{w(t)\}(\omega) = \frac{j\left(e^{-j\omega 0.6} + e^{-j\omega} - e^{-j\omega 0.7} - 1\right)}{\omega}. \tag{2}$$

The spectrum of the signal has infinite bandwidth. By the sampling theorem, to fully recover the signal, the sampling frequency must be twice the bandwidth. Therefore, there is no finite value sampling frequency that can fully recover the continuous signal. Graphically, this makes sense, because infinite resolution is required to capture the vertical edges of the signal.

The DFT is easily found using a fast Fourier transform algorithm (FFT). The signal was sampled at $f_s = 10$Hz on the range $t \in [0, 1]$. This yielded 11 samples of t. The DFT of the signal will have the same number of samples, on the range 0Hz to the sampling frequency of 10Hz. Due to aliasing, the DFT will always be symmetric over $f = \frac{f_s}{2}$. The plot of the DFT is seen in figure 4.

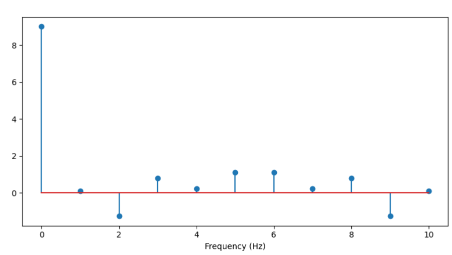

Fig. 4. DFT of $w(t)$ on $t \in [0, 1]$ with $f_s = 10$Hz

Likewise, the original signal can easily be recovered using an inverse fast Fourier transform (IFFT). Again the number of samples (11) does not change. We chose a sampling frequency of 10Hz, because it is able to capture the transition at $t = 0.6$ and its width of 0.1s. W.e plotted the IFFT using a bar plot starting at the edge, with each bar having a width of $T_s = \frac{1}{f_s}$, seen in figure 5. This method yielded a graph where the amplitude at each point matches the amplitude of the original signal. This method also worked to recover the original continuous signal for $f_s = 10n$Hz, $n \in \mathbb{N}^+$. For example, the plot is the same for $f_s = 20$Hz and $f_s = 30$Hz.

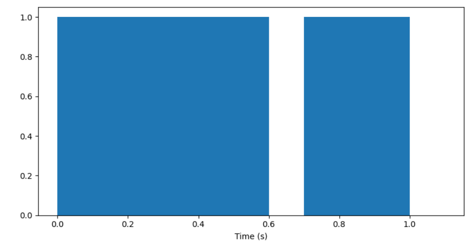

Fig. 5. The recovered signal from the DFT for $t \in [0, 1]$ with $f_s = 10$Hz

V. Synthesizing AM and FM Bandpass Signals and Analyzing Their Spectra

Modulation enables a message signal with frequency f_m, to be broadcast at a much higher carrier frequency f_c. When you tune a radio, the frequency being tuned to is the carrier frequency. The radio's job is to receive the high frequency signal and demodulate it to recover the original message.

Why add so many extra steps? Can't we just broadcast the message directly? The answer to the second question is sort of, but it is not a good idea for several of reasons. One reason is that message signals contain many of the same low frequencies. This means that if messages were broadcast directly, they would interfere with each other, making it nearly impossible to recover the desired signal. With the use of modulation, different transmitters can operate in a distinct band of the frequency spectrum. In doing so, the message signals no longer interfere and can be recovered with relative ease.

The two predominant forms of broadcast radio are amplitude modulation (AM) and frequency modulation (FM). We first examined an AM signal. AM signals are often of the form

$$s(t) = A_c(1 + A_m \cos(2\pi f_m t)) \cos(2\pi f_c t) \tag{3}$$

where A_c is the amplitude of the carrier signal, and A_m is the amplitude of the message signal. In our case, we used $f_c =$ 25kHz, $f_m =$ 5kHz, $A_c = 10$, and $A_m = 1$. The AM signal created was $s(t) = 10(1 + Am\cos(2\pi 5000t)) \cos(2\pi 25000t)$.

Next, we added noise to the signal using the same method as before. We compared the original signal to one with an SNR of 10 and another with an SNR of 20 and plotted them in figure 6. It appears that the noise affects the shape of the signal less where the slope is steeper, and more where it is shallower.

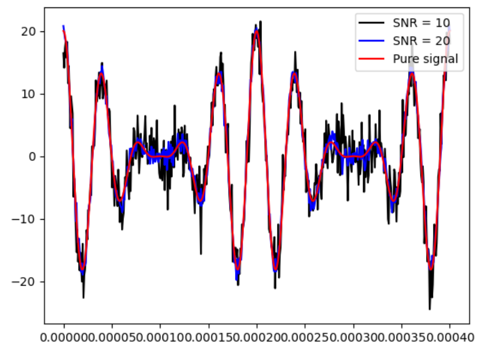

Fig. 6. An AM transmission at different SNRs

To better understand how the noise is affecting the signal, we analyzed the frequency spectrum of the signal. We obtained this by applying a fast Fourier transform (fft) to the samples of the signal. The resulting spectrum is seen in figure 7. The spectrum makes it clear that even though the time domain representation of the signal looks very different at an SNR of 10, the frequency components are not drastically affected.

Also note the three large spikes in the original spectrum. The large spike at 25000Hz corresponds to the contribution of the carrier frequency, and the two smaller spikes at 20000Hz and 30000Hz correspond to the contribution of the message frequency. The locations of these spikes is always centered at the carrier frequency, with the spectrum of the original signal centered at the carrier frequency rather than at the origin. Hence the smaller spikes at $f_c \pm f_m$.

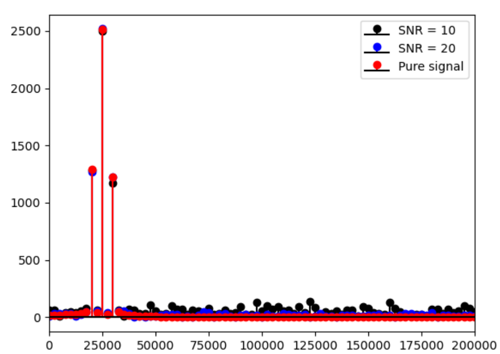

Fig. 7. The frequency spectrum an AM transmission at different SNRs

We also synthesized a bandpass FM signal with the same carrier and message frequencies as before. FM signals are of the form

$$s(t) = A_c \cos(2\pi f_c t + \beta_f A_m \sin(2\pi f_m t)) \tag{4}$$

where β_f is the frequency modulation index. In our tests we used $\beta_f = 10$. Therefore, our FM signal was $s(t) = 10\cos(2\pi 25000t + 10\sin(2\pi 5000t))$. Again we added noise at SNRs of 10 and 20, and plotted the results alongside the original signal, as seen in figure 8.

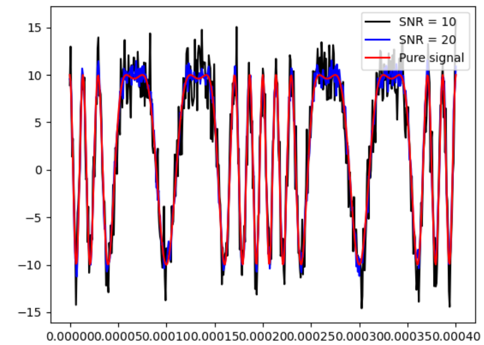

Fig. 8. Time domain plot of an FM signal at different SNRs

Finally, we examined the frequency spectrum of the FM signal. The spectrum of an FM signal will look like a clump of frequencies centered at the carrier frequency f_c. The width of this cluster depends on the frequency modulation index β_f. The larger β_f, the more spread out the spectrum becomes. With a large β_f like 10, as seen in figure 9, the spectrum is so spread out that it gets cut off. Changing β_f to something smaller like 2 will reduce the spread. Additionally, increasing the carrier frequency will prevent it from getting cut off. This may be one reason why FM stations have a carrier frequency near 100MHz.

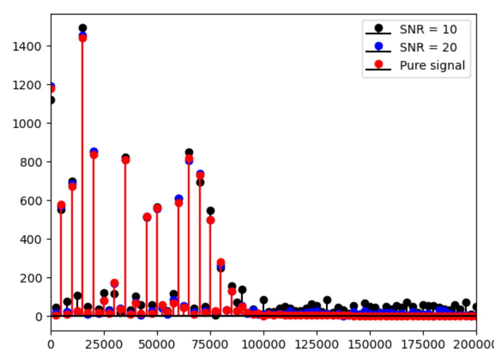

Fig. 9. The frequency spectrum of the FM signal at various SNRs

VI. Capturing and Analyzing the Spectra of One More Time

As an exercise in spectrum analysis, we selected a song called One More Time, and imported it into MATLAB. Then, we used the fast Fourier transform to obtain the song's frequency spectrum. As shown in 10, there is a much larger contribution from lower frequencies, and a very small contribution from frequencies higher than about 5kHz.

This makes sense for several reasons. Firstly, the range of human hearing is from about 20Hz to about 20kHz, which sets an upper bound on frequency. Another reason has to do with the three components of music: melody, harmony and rhythm.

Melody is the combination of note duration and pitch. A pattern of long notes will have a frequency component from the pitch of the note, but also a frequency component from the rate at which the notes are played. Since the duration of a note is on the order of a few Hertz at the fastest, the frequency of a string of notes will be quite low.

The contribution from rhythm has to do with the repetition of a string of notes or beats. For example, repeating a series of beats every five seconds will result in a frequency contribution at $\frac{1}{5} = 0.2\text{Hz}$.

Harmony will produce frequency components at regular increments, at what are called harmonics. Looking closely, there appears to be a regular spacing of spikes between about 1kHz and 3kHz in the spectrum.

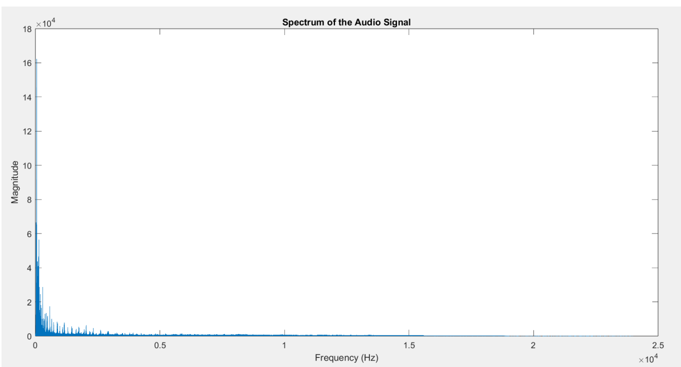

Fig. 10. Frequency spectrum of ”One More Time” by Daft Punk

VII. Conclusion

Over the course of this lab, there were a number of different experiments conducted and plots created to depict waveform synthesis and spectral analysis. These included plotting the frequency spectrum of a piece of music, generating waveforms with different SNRs, comparing different Fourier transforms, and comparing FM and AM signals. For the first portion of the lab involving generating different waveforms using different SNRs, we proved a greater SNR is increases the overall quality of a signal. This makes sense because if you corrupt a signal with noise and the amount of original signal is greater than that of the noise, the original signal will be easier to make out.

The second part of the lab involved comparing the CFT and the DFT of the same signal. After both were calculated and plotted, we learned that there is a difference not only in the values associated with the DFT and CFT, but they also have different applications when it comes to recovering signals.

For the third part of the lab, with spectral analysis of an FM and AM signal, the results of the tests showed that AM signals have a more compact and intuitive spectrum than FM signals. It also became apparent that the same amount of noise appears considerably more dramatic in the time domain than in the frequency domain. This is because both the signal and noise contribute to almost every point in the time domain, but in the frequency domain, the power of the signal is concentrated at specific points. It also became apparent that by using modulation, the spectrum of the message can be moved into a specific frequency band, so more people can transmit at the same time.

The final part of the lab involved plotting the frequency spectrum of a piece of music. This showed us how the frequency spectrum is similar for different signals because the lower frequencies are more common than the higher frequencies.

Overall, this lab assisted in understanding the topics of waveform synthesis and spectral analysis. We also gained a deeper understanding in how FM and AM signals are created, the reasons for creating them, and how noise affects their characteristics.