# Homework 3 - Aidan Sharpe

## 1

$$F(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-7x} & x \geq 0 \end{cases}$$

### a)

Find the probability density function of $X$:

$$f(x) = \frac{d}{dx}F(x) = \begin{cases} 0 & x < 0 \\ 7e^{-7x} & x \geq 0 \end{cases}$$

### b)

Verify that $f(x)$ is a valid density function:

$$\int_0^\infty f(x)dx \stackrel{?}{=} 1$$

$$\int_0^\infty 7e^{-7x}dx = \left[-e^{-7x}\Big|_0^\infty\right] = 0 - (-1) = 1$$

### c)

Find $P(X < 0.25)$:

$$F(0.25) = 1 - e^{-7(0.25)} = 0.826$$

### d)

Find $P(\frac{5}{60} < X < \frac{11}{60})$:

$$F\left(\frac{11}{60}\right) = 0.723$$

$$F\left(\frac{5}{60}\right) = 0.442$$

$$P\left(\frac{5}{60} < X < \frac{11}{60}\right) = 0.723 - 0.442 = 0.281$$

## 2

$$f(x) = \begin{cases} kx^{-3} & x > 1 \\ 0 & \text{elsewhere} \end{cases}$$

### a)

Find $k$, such that $f(x)$ is a valid density function:

$$\int_0^\infty f(x)dx \overset{!}{=} 1$$

$$\int_1^\infty kx^{-3}dx = \left[\frac{-k}{2x^2}\Big|_1^\infty\right] = 0 - \frac{-k}{2}$$

$$\therefore k = 2$$

### b)

Find $F(x)$:

$$F(x) = \int_{-\infty}^{x} f(t)dt$$

$$F(x) = \begin{cases} 1 - \frac{1}{x^2} & x > 1 \\ 0 & \text{elsewhere} \end{cases}$$

### c)

Find $P(X > 7)$:

$$P(X > 7) = F(7) = 0.98$$

### d)

Find $P(5 < X < 12)$:

$$P(X < 12) = F(12) = 0.993$$

$$P(X < 5) = 0.96$$

$$P(5 < X < 12) = F(12) - F(5) = 0.033$$

## 3

$$f(x) = \begin{cases} \frac{1}{5} & 1 < x < 6 \\ 0 & \text{elsewhere} \end{cases}$$

**a)**

Verify that $f(x)$ is a valid probability density function:

$$\int_{0}^{\infty} f(x)dx \stackrel{?}{=} 1$$

$$\frac{1}{5}(6-1) = 1$$

**b)**

Find $P(2.5 \leq X < 3)$:

$$P(2.5 \leq X < 3) = F(3) - F(2.5) = 0.4 - 0.3 = 0.1$$

**c)**

Find $P(X \leq 2)$:

$$P(X \leq 2) = F(2) = 0.4$$

**d)**

Find $F(x)$:

$$F(x) = \int_{-\infty}^{x} f(t)dt$$

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{(x-1)}{5} & 1 \leq x < 6 \\ 1 & x \geq 6 \end{cases}$$

## 4

A box contains 7 dimes and 5 nickels. Three coins are chosen. $T$ is their total value in cents.

**a)**

The set of all possible drawings of coins is:

$$\{DDD, DDN, DND, DNN, NDD, NDN, NND, NNN\}$$

Therefore the following are values for $T$:

$$T = \{30, 25, 20, 15\}$$

**b)**

Find the probability density function for $T$, $f(x)$:

$$P(T = 30) = \frac{\binom{7}{3}\binom{5}{0}}{\binom{12}{3}} = 0.159$$

$$P(T = 25) = \frac{\binom{7}{2}\binom{5}{1}}{\binom{12}{3}} = 0.477$$

$$P(T = 20) = \frac{\binom{7}{1}\binom{5}{2}}{\binom{12}{3}} = 0.318$$

$$P(T = 15) = \frac{\binom{7}{0}\binom{5}{3}}{\binom{12}{3}} = 0.046$$

$$f(x) = \begin{cases} 0.046 & x = 15 \\ 0.318 & x = 20 \\ 0.477 & x = 25 \\ 0.159 & x = 30 \end{cases}$$

**c)**

Find the CDF for $T$, $F(x)$:

$$f(x) = \begin{cases} 0 & x < 15 \\ 0.046 & 15 \leq x < 20 \\ 0.364 & 20 \leq x < 25 \\ 0.841 & 25 \leq x < 30 \\ 1 & x \geq 30 \end{cases}$$

## 5

| $x$ | $f(x)$ |
|---|---|
| 10 | 0.08 |
| 11 | 0.15 |
| 12 | 0.30 |
| 13 | 0.20 |
| 14 | 0.20 |
| 15 | 0.07 |

Determine the mean number of messages sent per hour

$$\sum_x xf(x) = 10(0.08) + 11(0.15) + 12(0.30) + 13(0.20) + 14(0.20) + 15(0.07) = 12.5$$

## 6

Find the expected value for each of the following probability density functions:

**a)**

$$f(x) = \begin{cases} x + \frac{1}{2} & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$E[x] = \int_0^1 \left( x + \frac{1}{2} \right) dx = \left[ \frac{x^2 + x}{2} \Big|_0^1 \right] = 1$$

**b)**

$$f(x) = \begin{cases} \frac{3}{x^4} & x \geq 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$E[x] \int_1^\infty \frac{3}{x^4} dx = \left[ \frac{-1}{x^3} \Big|_1^\infty \right] = 0 - (-1) = 1$$