EEMAGS Equation Sheet

I. Constants

$$\varepsilon_0 = 8.854 \times 10^{-12}$$

$$\mu_0 = 1.257 \times 10^{-6}$$

$$c = 3 \times 10^8$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\Omega \approx 120\pi$$

II. Fifth's Bullet Points

Find ε_r given λ and f:

$$\varepsilon_r = \frac{c^2}{\mu_r \lambda^2 f^2}$$

Find v_p given δ, α, and β given a good conductor:

$$v_p = \frac{\omega}{\beta}$$

Find P_{avg} given $\vec{H}(t)$ in air:
Find Γ in polar and VWSR given Z_L and Z_0:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\text{VWSR} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Find $\vec{E}(z,t)$ given $\vec{H}(z,t)$ in a lossless medium of μ_r and ε_r:
Find λ, ε_r, and $\vec{H}$ given $\vec{E}$:

$$\hat{H}_y = \frac{\hat{E}_x}{\eta}$$

$$\varepsilon_r = \frac{\beta^2}{\omega^2 \mu_r \mu_0 \varepsilon_0}$$

$$\lambda = \frac{v_p}{f}$$

Find Γ^2 and $\langle P \rangle$ from ε, lossless:
Find β_{air}, β_{material}, $\hat{\Gamma}$, $\hat{T}$ given $\vec{E}^i$, ε_r, μ_r, σ_r:
Find v_p, L, and Z_0 given ε_r and C, lossless:

$$v_p = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} = \frac{\omega}{\beta}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$L = \frac{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}{C}$$

Find C given Z_0, R_L, f, and VWSR:

$$C = \frac{L}{Z_0^2} - \frac{G}{j\omega} + \frac{R}{j\omega Z_0^2}$$

$$\text{VWSR} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$\Gamma = \frac{R_L - Z_0}{R_L + Z_0}$$

$$\omega = 2\pi f$$

Find VWSR, Γ, Z_{in} from λ, Z_0, Z_L, length:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\text{VWSR} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$\max[Z_{\text{in}}] = Z_0 \cdot \text{VWSR}$$

$$\min[Z_{\text{in}}] = \frac{Z_0}{\text{VWSR}}$$

Find Z_0, α, β, γ, λ given $Z_{\text{in, sc}}$ and $Z_{\text{in, oc}}$:

$$Z_0 = \sqrt{Z_{\text{in, sc}} Z_{\text{in, oc}}}$$

$$\gamma = \frac{1}{l} \operatorname{arctanh}\left(\sqrt{\frac{Z_{\text{in, sc}}}{Z_{\text{in, oc}}}}\right)$$

III. Boundary Conditions

Electric field boundary conditions:

$$\vec{E}_{T1} = \vec{E}_{T2}$$

$$\vec{D}_{N1} = \vec{D}_{N2}$$

$$\vec{E} = \vec{E}_N + \vec{E}_T$$

$$\vec{D} = \varepsilon \vec{E}$$

Magnetic field boundary conditions:

$$\vec{H}_{T1} = \vec{H}_{T2}$$

$$\vec{B}_{N1} = \vec{B}_{N2}$$

$$\vec{B} = \vec{B}_N + \vec{B}_T$$

$$\vec{B} = \mu \vec{H}$$

Normal vectors:

$$\vec{n} = \vec{E}_1 - \vec{E}_2$$

$$\vec{n} = \vec{H}_1 - \vec{H}_2$$

$$\hat{n} = \frac{\vec{n}}{\|\vec{n}\|}$$

IV. Laplace and Poisson

The Laplacian:

$$\vec{\nabla} \cdot (-\vec{\nabla} V) = -\vec{\nabla}^2 V = \frac{\rho}{\varepsilon_0}$$

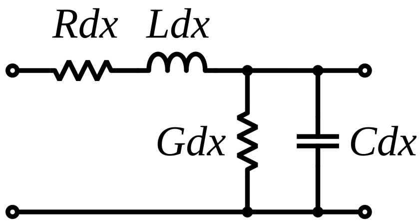
Laplace's Equation (charge free region):

$$\vec{\nabla}^2 V = 0$$

Poisson's Equation:

$$\vec{\nabla}^2 V = -\frac{\rho}{\varepsilon_0}$$

V. General Transmission Lines



$$\gamma = \alpha + j\beta = [(R + j\omega L)(G + j\omega C)]^{1/2}$$
$$= \frac{1}{l}\,\mathrm{arctanh}\left(\sqrt{\frac{Z_{\text{in, sc}}}{Z_{\text{in, oc}}}}\right)$$

The characteristic impedance:

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{Z_{\text{in, sc}} Z_{\text{in, oc}}}$$

$$Z_{\text{in}} = Z_0 \frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)}$$

VI. Voltage Standing Wave Ratio

$$\Gamma = \left.\frac{V_{\text{reflected}}}{V_{\text{incident}}}\right|_{z'=0} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\text{VWSR} = \frac{|V_{\text{max}}|}{|V_{\text{min}}|} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$v_p = \frac{\omega}{\beta}$$

$$P_{\text{avg}} = \frac{|V_0^+|}{2Z_0}(1 - |\Gamma_L|^2)$$

VII. Transmission Line Design

Lossless lines:

$$R = G = 0$$

$$\alpha = 0$$

$$\beta = \omega\sqrt{LC}$$

Distortionless lines:

$$\frac{R}{L} = \frac{G}{C}$$

$$\alpha = R\sqrt{\frac{C}{L}}$$

$$\beta = \omega\sqrt{LC}$$