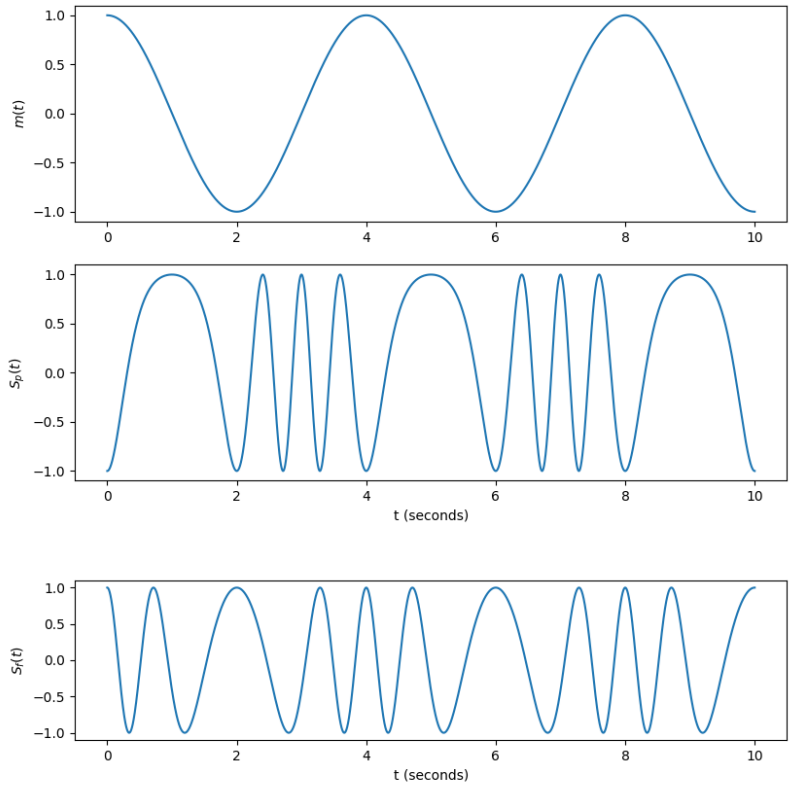
ECOMMS Homework 3 - Aidan Sharpe

Problem 1

A sinusoidal signal $m(t) = \cos(2\pi f_m t)$ is the input to an angle-modulated transmitter, $A_c = 1$, and the carrier frequency is $f_c = 1$ Hz and $f_m = f_c/4$. Plot $m(t)$, and the corresponding phase and frequency modulated signals, $S_p(t)$ and $S_f(t)$ respecively. $D_p = \pi$ and $D_f = \pi$.

```
f_c = 1
f_m = f_c/4

f_s = 8E3
T_s = 1/f_s

A_c = 1
D_p = np.pi
D_f = np.pi

omega_m = 2*np.pi*f_m
omega_c = 2*np.pi*f_c

# Time samples from to 10 seconds with a sampling frequency of 8kHz.
t = np.arange(0,10,T_s)

# The message signal
m = np.cos(omega_m * t)

# The phase modulated signal
S_p = np.cos(omega_c*t + D_p*m)

# The time integral of the mesage signal
M = np.sin(omega_m * t) / omega_m

# The frequency modulated signal
S_f = np.cos(omega_c*t + D_f*M)
```



Problem 2

An FM signal has sinusoidal modulation with a frequency of $f_m = 15$kHz and a modulation index of $\beta = 2.0$. Find the transmission bandwidth by using Carson's rule, and find the percentage of total FM signal power that lies within the Carson rule bandwidth.

```
# Modulation index
beta = 2.0

# Message frequency
f_m = 15E+3

# Transmission bandwidth
B_T = 2*(beta+1)*f_m
```

$\boxed{B_T = 90\text{kHz}}$

```
A_c = 1
n = np.arange(-3,4,1)

# Evaluate the Bessel function at values in Carson rule bandwidth
bessel_values = np.abs(sp.special.jv(n,beta))

P_C = 0.5 * A_c**2 * np.sum(bessel_values**2))
P = 0.5 * A_c**2
```

$\boxed{P_c = 0.9976P}$

Problem 3

A modulated RF waveform is given by $500\cos(\omega_c t + 20\cos(\omega_1 t))$, where $\omega_1 = 2\pi f_1$, $f_1 = 1\text{kHz}$, $\omega_c = 2\pi f_c$, and $f_c = 100\text{MHz}$.

3a

If the phase sensitivity $D_p = 100$ rad/V, find the mathematical expression for the corresponding phase modulation voltage $m(t)$. What is its peak value and frequency?

$m(t) = \frac{20\cos(\omega_1 t)}{D_p} = 0.2\cos(\omega_1 t)$

Peak value: 2×10^{-1}

Frequency: 1kHz

3b

If the frequency deviation constant $D_f = 10^6\text{rad/Vs}$, find the mathematical expression for the corresponding FM voltage $m(t)$. What is its peak value and its frequency?

$\theta(t) = 20\cos(\omega_1 t)$

$m(t) = \frac{1}{D_f}\frac{d\theta(t)}{dt} = -2 \times 10^{-5}\omega_1 \sin(\omega_1 t)$

Peak value: 2×10^{-5}

Frequency: 1kHz

3c

If the RF waveform appears across a 50Ω load, determine the average power and the PEP.

The average power is:

$$\langle s^2(t) \rangle = \frac{1}{T} \int_{-T/2}^{T/2} 500 \cos(\omega_c t + 20 \cos(\omega_1 t)) dt$$