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Homework 6

1.

a) Root Locus is not symmetrical about real axis ∴ not possible

b) There is one pole where the root locus is not originating from and there is a zero that has no termination ∴ not possible

c) The two zeros do not have termination ∴ not possible

d) Possible

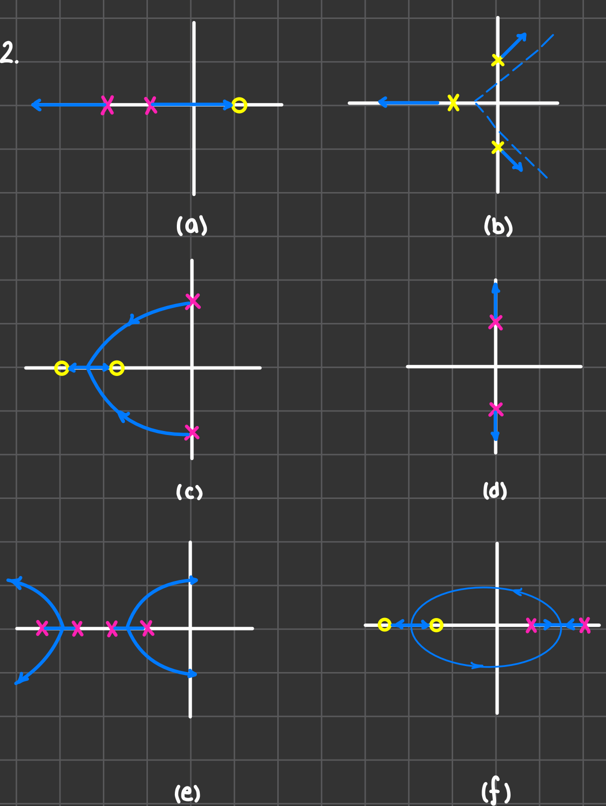
e) Not symmetrical about real axis ∴ not possible

f) Possible

g) Has single complex pole ∴ not possible

h) possible

2.



3. Infinite poles: 0, -5, -8

need 3 infinite zeros

$$\sigma_a = \frac{(0-5-8)}{3} = -4.33$$

$$\theta_a = \frac{(2k+1)180}{\#F_p - \#F_z} \rightarrow k=0$$

$$\frac{180}{3} = 60^\circ$$

$k=1$

$$\frac{3 \cdot 180}{3} = 180^\circ$$

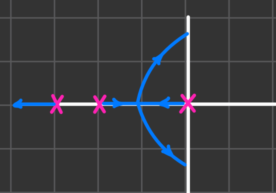
$k=2$

$$\frac{5 \cdot 180}{3} = 300^\circ$$

$$\sum_{1}^{m} \frac{1}{\sigma - z_i} = \sum_{1}^{n} \frac{1}{\sigma - R}$$

$$0 = \frac{1}{\sigma} + \frac{1}{\sigma+5} + \frac{1}{\sigma+8}$$

$\sigma = -2$



$$T(s) = \frac{K}{s^3 + 13s^2 + 40s + K}$$

s^3	1	40
s^2	13	K
s^1	$\frac{520-K}{13}$	0
s^0	K	0

row of zeros

$$\frac{520-K}{13} = 0 \rightarrow K = 520$$

$$13s^2 + K = 0$$

$$13s^2 = -520$$

$$s_{1,2} = j6.32$$

[redacted]

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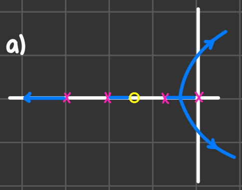
4

a)



b)

$$\sigma_a = \frac{\sum F_p - \sum F_z}{\# F_p - \# F_z} = \frac{(0-1-3-5)-(-2)}{4-1} = -2.33$$

$$\theta_a = \frac{(2K+1)180}{\# F_p - \# F_z} \longrightarrow K=0 \quad \frac{180}{4-1} = 60^\circ$$

$K=1$ $\quad \frac{3\cdot 180}{3} = 180^\circ$

$K=2$ $\quad \frac{5\cdot 180}{3} = 300^\circ$

$K=3$ $\quad \frac{7\cdot 180}{3} = 60^\circ$ $(420^\circ - 360^\circ = 60^\circ)$

c) $$T(s) = \frac{K(s+20)}{s^4 + 9s^3 + 23s^2 + (15+K)s + 2K}$$

s^4	1	23	2K
s^3	9	15+K	0
s^2	$23 - \frac{15+K}{9}$	2K	0
s^1	$15+K - \frac{18K}{23 - \frac{15+K}{9}}$	0	0
s^0	2K	0	0

$$15 + K - \frac{18K}{23 - \frac{15+K}{9}} = 0 \longrightarrow K = 61.69$$

d) $$K = \frac{1}{|G(s)H(s)|} = \left|\frac{s(s+1)(s+3)(s+5)}{(s+2)}\right| = \left|\frac{(-0.5)(0.5)(2.5)(4.5)}{(1.5)}\right| = 1.875$$

5.

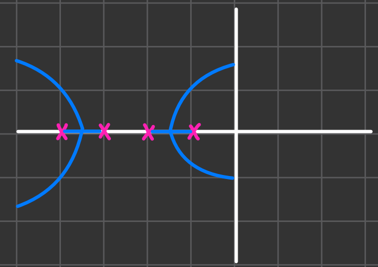
$$K = \frac{1}{|G(s)H(s)|} = \left|\frac{s(s+3)(s+6)}{s+\alpha}\right| = \left|\frac{(-1+j100)(2+j100)(5+j100)}{(-1+\alpha)+j100}\right| = \frac{10\sqrt{1003001290l}}{\sqrt{a^2+2a+1001}} \quad \text{eqn 1}$$

$$T(s) = \frac{K(s+\alpha)}{s^3+9s^2+(K+18)s+\alpha K} \longrightarrow s^3+9s^2+(K+18)s+\alpha K = 0 \quad \text{eqn 2.}$$

Solving sys of eqns.

$\alpha = 7 \quad K = 9997$

6.



a)

$$\sigma_a = \frac{\sum F_p - \sum F_z}{\# F_p - \# F_z} = \frac{(-1-2-3-4)}{4} = -2.5$$

$$\theta_a = \frac{(2K+1)180}{\# F_p - \# F_z} \longrightarrow K=0 \quad \frac{180}{4} = 45^\circ$$

$K=1$ $\quad \frac{3\cdot 180}{4} = 135^\circ$

$K=2$ $\quad \frac{5\cdot 180}{4} = 225^\circ$

$K=3$ $\quad \frac{7\cdot 180}{4} = 315^\circ$

b) $$\sum_1^m \frac{1}{\sigma + z_i} = \sum_1^n \frac{1}{\sigma + R}$$

$$0 = \frac{1}{\sigma+1} + \frac{1}{\sigma+2} + \frac{1}{\sigma+3} + \frac{1}{\sigma+4}$$

$\sigma = -3.61, -2.5, -1.38$

Breakaway point : -3.61 & -1.38 (the only two on root locus)

c) $s^4 + 10s^3 + 35s^2 + 50s + (24+K)$

s^4	1	35	24+K
s^3	10	50	0
s^2	30	24+K	0
s^1	$50 - \frac{24+K}{3}$	0	0
s^0	24+K	0	0

$$50 - \frac{24+K}{3} > 0 \longrightarrow 0 < K < 126$$

d) $\phi = 180 - \cos^{-1}(0.7) = 134.43^\circ$

134.43° from origin corresponds to $-0.992 + j1.01$.

$K = |(s+1)(s+2)(s+3)(s+4)|$

If $s = -0.992 + j1.01$

$K \approx 10.3$

6 cont.

e) $\angle KG(S)H(S) = -180$

$$\tan^{-1}\left(\frac{5.5}{\alpha}\right) - \tan^{-1}\left(\frac{5.5}{4}\right) - \tan^{-1}\left(\frac{5.5}{3}\right) - \tan^{-1}\left(\frac{5.5}{2}\right) = 180$$

$$\tan^{-1}\left(\frac{5.5}{\alpha}\right) - 53.97^\circ - 61.39^\circ - 70.02^\circ - 79.70^\circ = 180$$

$$\tan^{-1}\left(\frac{5.5}{\alpha}\right) = 445.08^\circ$$

$$\frac{5.5}{\alpha} = \tan(445.08^\circ)$$

$$\alpha = 0.473$$

f)

$$1 + KG(S)H(S) = S^4 + 10S^3 + 35S^2 + (50+K)S + (24+0.473K)$$

S^4	1	35	$24 + 0.473K$
S^3	10	$50+K$	0
S^2	$30 - \frac{K}{10}$	$24+0.473K$	0
S^1	$50 + K - \frac{240+4.73K}{30-\frac{K}{10}}$	0	0
S^0	$24 + 0.473K$	0	0

$$50 + K - \frac{240 + 4.73K}{30 - \frac{K}{10}} = 0 \longrightarrow 0 < K < 252.58$$

g) Adding a zero to the system increased the range for K, meaning the sysmem would be stable for a greater range of gain. Additionally, the rise time and settling time of the system were reduced.