15.1

1 $f(x,y) = xy$ $\quad \int_1^3\int_1^{2.5} xy\,dy\,dx \quad \frac{xy^2}{2}\Big|_1^{2.5} \quad \frac{6.25x}{2} - \frac{x}{2} \quad \int_1^3 \frac{5.25x}{2}\,dx$

$2.625\left(\frac{x}{2}\Big|_1^3\right. \quad \frac{3}{2} - \frac{1}{2} = 1 \quad \boxed{2.625}$

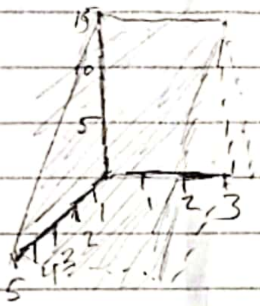
3 $\int_1^4\int_1^3 2x + 1y\,dy\,dx \quad 2xy + \frac{y^2}{2}\Big|_1^3 \quad \left(6x + \frac{9}{2}\right) - \left(2x + \frac{1}{2}\right)$

$\int_1^4 4x + 4\,dx \quad 2x^2 + 4x\Big|_1^4 \quad 32 + 16 - 6 = \boxed{42}$

9 $\int_0^5\int_0^3 (15 - 3x)\,dy\,dx \quad \int_0^5 15y - 3xy\Big|_0^3 dx \quad 45 - 9x \quad \cancel{45x} \quad 9\left(5x - \frac{x^2}{2}\Big|_0^5\right)$

$25 - \frac{25}{2} = \frac{25}{2} \cdot 9$

$\boxed{\frac{225}{2}}$



19 $\int_1^3\int_0^2 x^3 y\,dy\,dx \quad \frac{x^3y^2}{2}\Big|_0^2 \quad \int_1^3 2x^3\,dx \quad \frac{x^4}{2}\Big|_1^3 \quad \frac{81}{2} - \frac{1}{2} = \boxed{40}$

21 $\int_4^9\int_{-3}^8 1\,dx\,dy \quad x\Big|_{-3}^8 \quad 8 + 3 = 11y\Big|_4^9 \quad 99 - 44 = \boxed{55}$

27 $\int_0^1\int_0^2 (x + 4y^3)\,dx\,dy \quad \frac{x^2}{2} + 4y^3x\Big|_0^2 \quad 2 + 8y^3 \quad 2\left(y + y^4\Big|_0^1\right) = \boxed{4}$

33 $\int_0^4\int_0^5 \frac{1}{\sqrt{x+y}}\,dy\,dx \quad (x+y)^{-\frac{1}{2}} \quad 2(x+y)^{\frac{1}{2}}\Big|_0^5$

$2\sqrt{x+5} - 2\sqrt{x}$

$2\int_0^4 \sqrt{x+5} - \sqrt{x}\,dx \quad 2\left(\frac{2}{3}(x+5)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}}\Big|^4\right.$

$2\left(\left(18 + \frac{16}{3}\right) - \frac{10\sqrt{5}}{3}\right) = \boxed{10.42}$

15.1 (continued)

37 $\int_1^3\int_{-2}^4 \frac{x}{y}\,dx\,dy$ $\quad \frac{x^2}{2y}\Big|_{-2}^4$ $\quad \frac{16}{2y}-\frac{4}{2y}=\int_1^3 \frac{6}{y}\,dy$ $\quad 6\int_1^3 \frac{1}{y}\,dy$ $\quad 6\left(\ln(y)\Big|_1^3\right)$

$\boxed{6\ln 3}$

41 $\int_0^2\int_0^{\frac{\pi}{4}} e^x \sin y\,dy\,dx$ $\quad -e^x\cos y\Big|_0^{\frac{\pi}{4}}$ $\quad -e^x\frac{\sqrt{2}}{2}-e^0$ $\quad e^x\left(1-\frac{\sqrt{2}}{2}\right)\Big|_0^2$

$\boxed{(e^2-1)(1-\frac{\sqrt{2}}{2}) = 1.87}$

15.2

3 $\int_0^1\int_0^{1-x^2} y\,dy\,dx$ $\quad \frac{y^2}{2}\Big|_0^{1-x^2}$ $\quad \frac{(1-x^2)^2}{2}$ $\quad \frac{(1-x^2)^3}{-12}$

$\frac{x^5}{10} - \frac{x^3}{3} + \frac{1}{2}x \Big|_0^1$ $\quad \frac{1}{10}-\frac{1}{3}+\frac{1}{2} = \frac{3}{30}-\frac{10}{30}+\frac{15}{30} = \boxed{\frac{8}{30}}$

5 $\int_0^4\int_{2-\frac{x}{2}}^{2} x^2 y\,dy\,dx$ $\quad \frac{xy^2}{2}\Big|_{2-\frac{x}{2}}^{2}$ $\quad \int_0^4 2x^2 - \frac{x^2(2-\frac{x}{2})^2}{2}\,dx = \int_0^4 x^3 - \frac{x^4}{4}\,dx$ $\quad \frac{x^4}{4} - \frac{x^5}{40}\Big|_0^4$

$64 - \frac{1024}{40} = \boxed{\frac{192}{5}}$

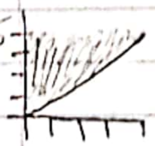
11 $\int_1^2\int_0^{\sqrt{4-x^2}} \frac{y}{x}\,dA$ $\quad \frac{y^2}{2x}\Big|_0^{\sqrt{4-x^2}}$ $\quad \frac{4-x^2}{2x} = \int_1^2 \frac{2}{x} - \frac{x}{2}\,dx$ $\quad 2\ln x - \frac{x^2}{4}\Big|_1^2$ $\quad \boxed{2\ln 2 - \frac{3}{4}}$

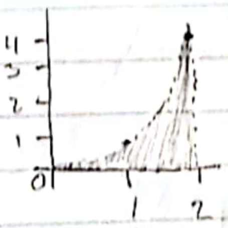
$(2\ln 2 - 1) - (-\frac{1}{4})$

13 $\int_0^{\pi}\int_0^2 (r\cos\theta + r\sin\theta)\,r\,dr\,d\theta$ $\quad \frac{r^3}{3}(\cos\theta+\sin\theta)\Big|_0^2$ $\quad \frac{8}{3}(\cos\theta+\sin\theta)$

$\frac{8}{3}\left((\sin\theta - \cos\theta)\Big|_0^{\pi}\right)$ $\quad \frac{8}{3}(2)$ $\quad \boxed{\frac{16}{3}}$

15 $\int_0^1\int_{-\sqrt{1-x}+1}^{2x-x^2} x\,dy\,dx$ $\quad y = 2x - x^2$, $y = -\sqrt{1-x}+1$ $\quad xy\Big|_{-\sqrt{1-x}+1}^{2x-x^2}$ $\quad \int_0^1 (2x^2-x^3) + (x\sqrt{1-x} - x)\,dx = \boxed{\frac{11}{60}}$

25  $\quad \int_0^4\int_0^y f(x,y)\,dx\,dy$

29 $\int_0^4\int_{\sqrt{y}}^{2} \sqrt{4x^2+5y}\,dx\,dy$ 

45 $\int_0^2\int_0^{4-x^2}\int_0^{40-10y} 1\,dz\,dy\,dx$ $\quad z\Big|_0^{40-10y}$ $\quad 40y - 5y^2\Big|_0^{4-x^2}$ $\quad \boxed{128}$

15.2 (continued)

47 $\int_0^{2\sqrt{2}}\int_{x^2}^{8-x^2}\int_y^{16-y} z\, dz\,dy\,dx$ $\quad z\Big|_y^{16-y}$ $\quad 16y-y^2\Big|_{x^2}^{8-x^2}$ $\quad 128-16x^2-(8-x^2)^2-(16x^2-x^4)$

$\boxed{\frac{128\sqrt{2}}{3}}$

$64-16x^2$

$16\int_0^{2\sqrt{2}} 4-x^2\,dx$ $\quad 4x-\frac{x^3}{3}\Big|_0^{2\sqrt{2}}$ $\quad 8\sqrt{2}-\frac{16\sqrt{2}}{3}$

$16\left(\frac{8\sqrt{2}}{3}\right)$

55 $\int_0^{\pi}\int_0^{1}\int_0^{y^2\sin x} 1\, dz\,dy\,dx$ $\quad$ Area of region below surface: π

$y^2\sin x$ $\quad \frac{y^3}{3}\sin x\Big|_0^1$ $\quad \frac{\sin x}{3}$ $\quad \frac{-\cos x}{3}\Big|_0^{\pi} =$

Volume of region $= \frac{2}{3}$ $\quad$ Average height $= \frac{2/3}{\pi} = \boxed{\frac{2}{3\pi}}$

59 maximum value of $\sqrt{x^3+1}$ on A is 3

$\int_0^2\int_{-\frac{1}{4}}^{\frac{1}{8}} 3\, dA$ $\quad 3y$ $\quad \frac{6}{8}x\Big|_0^2$ $\quad \frac{12}{8} = \boxed{\frac{3}{2}}$

since $3 \geq \sqrt{x^3+1}$ everywhere on the interval, then volume $< \frac{3}{2}$

15.3

1 $\int_0^2\int_2^4\int_0^4 xz+yz^2 \, dz\,dy\,dx$ $\frac{xz^2}{2}+\frac{yz^3}{3}\Big|_0^4$ $8x+\frac{64}{3}y$ $8xy+\frac{32}{3}y^2\Big|_2^4$

$16x+128$ $8x^2+128x\Big|_0^2$ $32+256=\boxed{288}$

5 $\int_0^1\int_0^3\int_0^3 xy-y^2+yz-xz \, dz\,dy\,dx$ $xyz-y^2z+\frac{yz^2}{2}-\frac{xz^2}{2}\Big|_0^3$

$\frac{3xy^2}{2}-y^3+\frac{9y^2}{4}-\frac{9}{2}xy\Big|_0^3$ $3xy-3y^2+\frac{9y}{2}-\frac{9x}{2}$

$\frac{27x}{2}-27+\frac{81}{4}-\frac{27x}{2}$ $\frac{81}{4}-27$ $\frac{-27}{4}x\Big|_0^1$ $\boxed{\frac{-27}{4}}$

9 $\int_0^1\int_0^x\int_y^x x+y \, dz\,dy\,dx$ $xz+yz\Big|_y^x$ $x^2+xy-(y^2+xy)$ x^2-y^2 $x^2y-\frac{y^3}{3}\Big|_0^x$ $x^3-\frac{x^3}{3}$

$\frac{2}{3}x^3$

$\frac{1}{3}\left(\frac{x^4}{2}\Big|_0^1\right)=\boxed{\frac{1}{6}}$

13 $\int_0^1\int_0^{1-x}\int_0^{1-(x+y)} e^z \, dz\,dy\,dx$ $e^{(1-(x+y))}-e^0$ $\frac{e}{e^xe^y}-1$ $\frac{-e}{e^xe^y}-y\Big|_0^{1-x}$ $\boxed{e-\frac{5}{2}}$

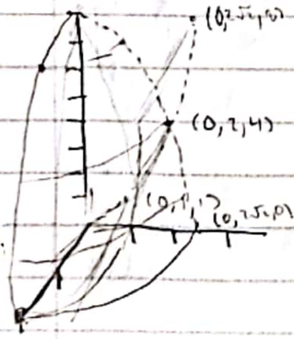
$z \le 1-(x+y)$ $-e^{(1-x)}+\frac{x^2}{2}-2x\Big|_0^1$ $-1+\frac{1}{2}-2=-\frac{5}{2}$

$y \le 1-x$ $\frac{-5}{2}-\,-e$

$x \le 1$

17 $\int_0^2\int_0^{\sqrt{8-2x^2}}\int_{y^2}^{8-2x^2-y^2} x \, dz\,dy\,dx$ $xz\Big|_{y^2}^{8-2x^2-y^2}$ $(8x-2x^3-y^2x)-(xy^2)$ $8xy-2x^3y-\frac{2y^3x}{3}\Big|_0^{\sqrt{8-x^2}}$

$\int_0^2\int_0^{\sqrt{8-2x^2}} 8x-2x^3-2y^2x \, dy\,dx$



$\boxed{\frac{64\sqrt{2}}{15}}$

15.3 (continued)

21 $z = 1 - (x+y)$

$z = \frac{1-(x+y)}{2}$

$y = 1 - x$

$y =$

$2z =$

$$\int_0^1 \int_0 \int$$

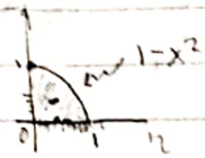
23 $\iiint xz \, \partial V$

15.5

1 $\int_0^1\int_0^2 2x^2+y^2\,dy\,dx$ $\quad 2x^2y+\frac{y^3}{3}\Big|_0^2$ $\quad 4x^2+\frac{8}{3}$ $\quad 4\int_0^1 x^2+\frac{2}{3}\,dx$ $\quad 4\left(\frac{x^3}{3}+\frac{2x}{3}\Big|_0^1\right)$ $\quad \boxed{4}$

$\frac{1}{3}+\frac{2}{3}=1$

11



$\bar{x}=\frac{1}{A}\int_0^1\int_0^{1-x^2} x\,dy\,dx$ $\quad \bar{y}=\frac{1}{A}\int_0^1\int_0^{1-x^2} y\,dy\,dx$ $\quad \frac{y^2}{2}$ $\quad \frac{(1-x^2)^2}{2}$

$A=\int_0^1 1-x^2\,dx$ $\quad x-x^3$ $\quad \frac{x^2}{2}-\frac{x^4}{4}\Big|_0^1=\left(\frac{1}{4A},\frac{4}{15A}\right)$ $\quad \frac{x^5}{10}-\frac{x^3}{3}+\frac{x}{2}\Big|_0^1$ $\quad \frac{1}{10}-\frac{1}{3}+\frac{1}{2}$

$\frac{13}{30}-\frac{10}{30}=\frac{4}{15}$

$x-\frac{x^3}{3}\Big|_0^1$

$1-\frac{1}{3}=\frac{2}{3}$ $\quad \boxed{\left(\frac{3}{8},\frac{4}{10}\right)}$

23 $\bar{x}=0$ $\quad \bar{y}=0$ $\quad \bar{z}=\int_0^{2\pi}\int_0^1\int z$

17.1

1. $\oint_C xy\,dx + y\,dy \quad F = \langle xy, y\rangle \qquad 0 \le t \le 2\pi$

$r(t) = \langle \cos t, \sin t\rangle$

$\int_0^{2\pi} \cos t \cdot \sin t \cdot \sin t + \sin t \cdot (-\cos t)\,dt \qquad \frac{\sin^3 t}{3} - \frac{\sin^2 t}{2}\Big|_0^{2\pi} = \boxed{0}$

$\cos t \sin^2 t - \sin t \cos t$

$\iint_D (0 - x)\,dA \qquad \int_0^{2\pi}\int_0^1 -r^2\cos\theta\,dr\,d\theta \qquad \frac{-r^3\cos\theta}{3}\Big|_0^1 \qquad \frac{-1}{3}\cos\theta$

$\boxed{0 = 0 \checkmark} \qquad \frac{-1}{3}\left(\sin\theta\Big|_0^{2\pi}\right) = \boxed{0}$

3 $\oint y^2dx\ \ x^2dy \qquad F = \langle y^2, x^2\rangle$

$\int_0^1\int_0^1 2x - 2y\,dy\,dx \qquad 2xy - y^2\Big|_0^1 \qquad 2x - 1 \qquad x^2 - x\Big|_0^1 = \boxed{0}$

7 $\oint_C x^2y\,dx \qquad F = \langle x^2y, 0\rangle$

$\iint_D 0 - x^2\,dA = \int_0^{2\pi}\int_0^1 -(r(\cos\theta))^2\,r\,dr\,d\theta \qquad -r^3\cos^2\theta$

$\frac{-r^4\cos^2\theta}{4}\Big|_0^1 \quad -\frac{1}{4}\int_0^{2\pi}\cos^2\theta\,d\theta$

$\frac{-1}{4}\left(\frac{\sin\theta\cos\theta}{2} + \frac{\theta}{2}\right|_0^{2\pi} \boxed{-\frac{\pi}{4}}$

9 $\oint F \cdot dr \qquad F = \langle x^2, x^2\rangle \qquad \int_0^1\int_{x^2}^x 2x - 0\,dy\,dx \qquad 2xy\Big|_{x^2}^x \qquad \int_0^1 2x^2 - 2x^3\,dx$

$\boxed{\frac{1}{6}}$

$2\int_0^1 x^2 - x^3\,dx$

$\frac{x^3}{3} - \frac{x^4}{4}\Big|_0^1 \qquad \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

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17.2

1 $F=\langle 2xy, x, y+z\rangle \quad z=1-x^2-y^2 \quad x^2+y^2\leq 1$

$\oint_S 2xy\,dx + x\,dy + (y+z)\,dz$

$r(t)=\langle x(t), y(t), f(x(t),y(t))\rangle$

$x(t)=\cos(t)$

$y(t)=\sin(t)$

$\int_a^b 2(x(t)y(t))x'(t) + x(t)y'(t) + (y(t)+z)z'\,dt \quad z=f(x(t),y(t))=1-\cos^2 t-\sin^2 t$

$\left.\frac{\sin t\cos t}{2} - \frac{2\cos^3 t}{3} - \frac{t}{2}\right|_{2\pi}^{0}$

$\boxed{\pi}$

$\text{curl}(F)=\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy & x & y+z \end{vmatrix} = \langle 2,0,1-2x\rangle$

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