# Aidan Sharpe - Homework 2

## 1.

a) Point $O$ exists at the origin, and point $B$ exists at $(16, 0)$. A 5kN force, $\vec{F}$, makes an angle of $\frac{5\pi}{6}$ with the $\hat{x}$ direction at $(12, -15)$.

### a) Find the moment, $M_O$, about point $O$.

$$M_O = \vec{r} \times \vec{F}$$

$$\vec{r} = (12 - 0)\hat{x} + (-15 - 0)\hat{y} = 12\hat{x} - 15\hat{y}$$

$$\vec{F} = 5000\cos\left(\frac{5\pi}{6}\right)\hat{x} + 5000\sin\left(\frac{5\pi}{6}\right)\hat{y} = -4330.13\hat{x} + 2500\hat{y}$$

$$\vec{r} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 12 & -15 & 0 \\ -4330.13 & 2500 & 0 \end{vmatrix} = -34951.91\hat{z}$$

$$M_O = -34951.91\hat{z}[\text{N m}]$$

### b) Find the moment, $M_B$, about point $B$

$$M_B = \vec{r} \times \vec{F}$$

$$\vec{r} = (12 - 16)\hat{x} + (-15 - 0)\hat{y} = -4\hat{x} - 15\hat{y}$$

$$\vec{F} = 5000\cos\left(\frac{5\pi}{6}\right)\hat{x} + 5000\sin\left(\frac{5\pi}{6}\right)\hat{y} = -4330.13\hat{x} + 2500\hat{y}$$

$$\vec{r} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -4 & -15 & 0 \\ -4330.13 & 2500 & 0 \end{vmatrix} = -74951.91\hat{z}$$

$$M_B = -74951.91\hat{z}[\text{N m}]$$

## 2

A 250N force is applied at an angle $\frac{5\pi}{12}$ with respect to the $\hat{x}$ direction at a distance 30mm above and 200mm to the right of the center of a bolt. Find the moment, $M_B$, about the center of the bolt.

$$M_B = \vec{r} \times \vec{F}$$

Convert distance to meters:

$$\vec{r} = 0.2\hat{x} + 0.03\hat{y}$$

$$\vec{F} = 250\cos\left(\frac{5\pi}{12}\right)\hat{x} + 250\sin\left(\frac{5\pi}{12}\right)\hat{y} = 64.70\hat{x} + 241.48\hat{y}$$

$$\vec{r} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0.2 & 0.03 & 0 \\ 64.70 & 241.48 & 0 \end{vmatrix} = 46.36\hat{z}$$

$$M_B = 46.36\hat{z}[\text{N m}]$$

## 3

Consider a bar made up of two segments, $\overline{AB}$ and $\overline{BC}$, each with length, 1.6m. Segment $\overline{AB}$ makes an angle $\frac{\pi}{2}$ with the $\hat{x}$ direction, and segment $\overline{BC}$ makes an angle $\frac{3\pi}{4}$ with the $\hat{x}$ direction. A 30N force, $\vec{P}$, is applied perpendicular to $\overline{BC}$. Determine the moment, $M_B$, about point $B$, and $M_A$ about point $A$.

### a) Find the moment, $M_B$, about point $B$

$$M_B = \vec{r} \times \vec{F}$$

$$\vec{r} = 1.6\cos\left(\frac{3\pi}{4}\right)\hat{x} + 1.6\sin\left(\frac{3\pi}{4}\right)\hat{y} = -1.13\hat{x} + 1.13\hat{y}$$

$$\vec{F} = 30\cos\left(\frac{\pi}{4}\right)\hat{x} + 30\sin\left(\frac{\pi}{4}\right)\hat{y} = 21.21\hat{x} + 21.21\hat{y}$$

$$\vec{r} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -1.13 & 1.13 & 0 \\ 21.21 & 21.21 & 0 \end{vmatrix} = -47.93\hat{z}$$

$$M_B = -47.93\hat{z}[\text{N m}]$$

### b) Find the moment, $M_A$, about point $A$

$$M_A = \vec{r} \times \vec{F}$$

$$\vec{r} = 1.6\cos\left(\frac{3\pi}{4}\right)\hat{x} + \left[1.6 + 1.6\sin\left(\frac{3\pi}{4}\right)\right]\hat{y} = -1.13\hat{x} + 2.73\hat{y}$$

$$\vec{F} = 30\cos\left(\frac{\pi}{4}\right)\hat{x} + 30\sin\left(\frac{\pi}{4}\right)\hat{y} = 21.21\hat{x} + 21.21\hat{y}$$

$$\vec{r} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -1.13 & 2.73 & 0 \\ 21.21 & 21.21 & 0 \end{vmatrix} = -81.87\hat{z}$$

$$M_A = -81.87\hat{z}[\text{N m}]$$

## 4

Consider an arm holding a ball and the moments acting on the elbow. The forearm makes an angle, $-35°$ with the $\hat{x}$ direction. The ball weighs 8lbs, and its center of gravity is a distance of 13 inches away in the $\hat{x}$ direction. The forearm weighs 5lbs, and its center of gravity is 6 inches away along the forearm. A third, balancing force of tension acts 2 inches down the forearm. Find the force of tension, $\vec{T}$, such that the moment about the elbow, $M_O$, is 0.

$$M_O = (\vec{r}_T \times \vec{T}) + (\vec{r}_G \times \vec{G}) + (\vec{r}_A \times \vec{A}) = 0$$

Calculate radial vectors:

$$\vec{r}_G = 6\cos(-35°)\hat{x} + 6\sin(-35°)\hat{y} = 4.91\hat{x} - 3.44\hat{y}$$

$$\vec{r}_A = 13\hat{x} + 13\tan(-35°)\hat{y} = 13\hat{x} - 9.10\hat{y}$$

$$\vec{r}_T = 2\hat{x} + 2\tan(-35°)\hat{y} = 2\hat{x} - 1.4\hat{y}$$

Assign force vectors:

$$\vec{G} = -5\hat{y}$$

$$\vec{A} = -8\hat{y}$$

Evaluate known cross products:

$$\vec{r}_G \times \vec{G} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 4.91 & -3.44 & 0 \\ 0 & -5 & 0 \end{vmatrix} = -24.55\hat{z}$$

$$\vec{r}_A \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 13 & -9.10 & 0 \\ 0 & -8 & 0 \end{vmatrix} = -104\hat{z}$$

Combine all knowns:

$$0 = (\vec{r}_T \times \vec{T}) - 24.55\hat{z} - 104\hat{z}$$

$$\therefore \vec{r}_T \times \vec{T} = 128.55\hat{z}$$

Since $\vec{T}$ only acts in the $\hat{y}$ direction:

$$128.55 = r_{T_x}T_y - r_{T_y}T_x = (2)T_x - (-1.4)(0)$$

$$\therefore 128.55 = (2)T_y$$

$$\therefore T_y = 64.28$$

$$\vec{T} = 64.28\hat{y}[\text{lb in}]$$