Laplace and Poisson

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1 Laplace's Equation for Potential with Boundary Conditions

This question involves taking many values in between zero and five in order to get a more accurate graph. The program cemLaplace03.m is used by first setting five grid points. Then, it sets a boundary potential of one hundred volts and a tolerance of 0.1. It sets the voltage zeroes at five and one, and uses them to perform calculations. It divides the sum of squares by the number of iterations in order to find the exact solution and electric field. As shown in 1, the voltage decreases almost linearly from one hundred to zero across a distance of five meters. The electric field remains constant at twenty volts per meter over the distance of five meters.

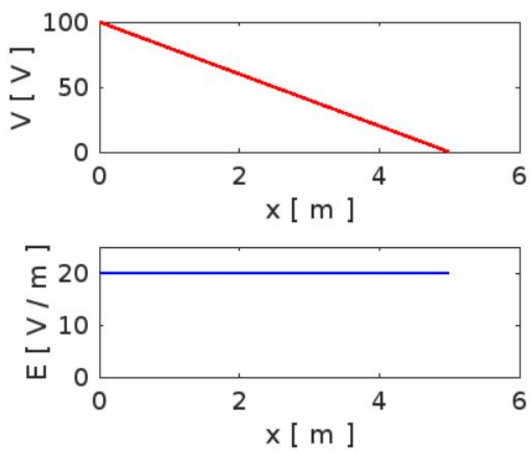

Figure 1: Electric potential over distance, and electric field over distance

2 Two Parallel Semi-infinite Metal Plates

This question utilizes the provided program cemLaplace02.m. This program works by taking a hundred and one grid points between zero and four in the x-axis and negative one and one in the y-axis. It sets the tolerance as 0.001, and the boundaries for the electric potential. Then, it divides the sum of squares by the number of iterations. As depicted in 2, the electric potential decreases over the distance.

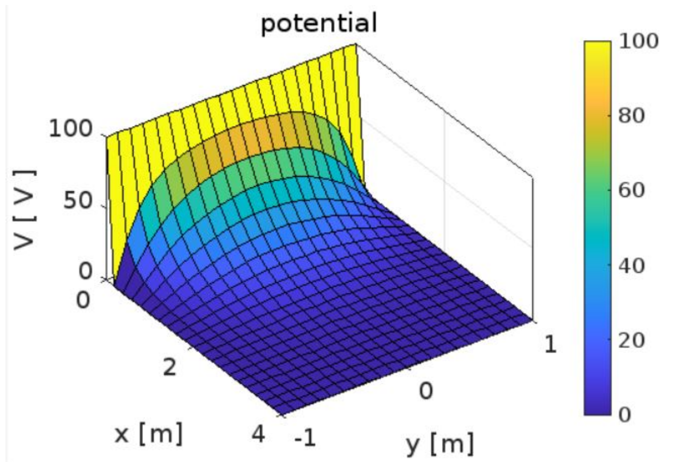

Figure 2: Electric potential (voltage) over distance

The electric field is computed using gradient. It decreases over distance, as seen in 3. The electric field is much stronger closer to the electric potential.

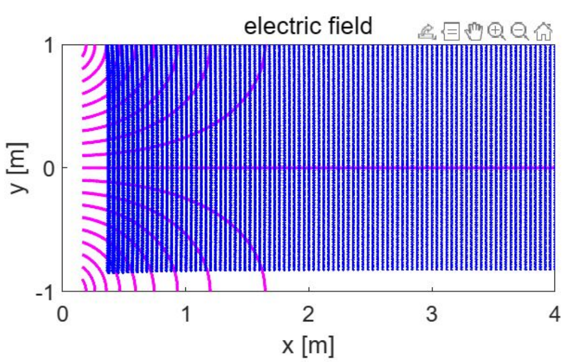

Figure 3: Electric field over distance

3 Investigating Relaxation Method to Solve Laplace's Equation

Along with assistance from a program titled cemLaplace01.m, one can make a rectangle of dimensions two meters by one meter. The program sets the number of grid points to be seven along the x-axis and y-axis and setting the rectangle to be from zero to two in the x-direction and zero to one in the y-direction. The tolerance is set to be 0.9. The boundaries are ten volts and all interior grid points are zero volts. This program also takes the sum of all of the differences in sum of squares divided by the number of iterations.

Before computing, it can be assumed that the electric electric field is strongest when closest to the greatest electric potential, which is ten volts. As depicted by 4, the electric potential is greatest at the edges of the rectangle. In order to achieve an accuracy of better than one percent, one should increase the number of iterations. 4 shows a graph with seventy-five iterations. This is more accurate than the graph with fifty iterations.

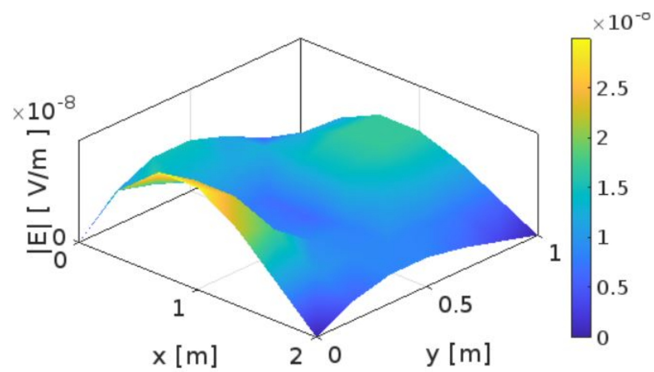

Figure 4: Electric potential across distance

4 Adjusting for Different Potentials on Boundaries

This question is very similar to the previous question. It utilizes the same program, but includes changing a few values for the boundary conditions. For the top of the rectangle, the voltage is set to negative ten volts and the bottom is set to negative five volts. The left side of the rectangle is twenty volts and the right side is ten volts. The electric field depicted in 5 shows the strength of the field when closer to the center, and weaker farther away from electric potential.

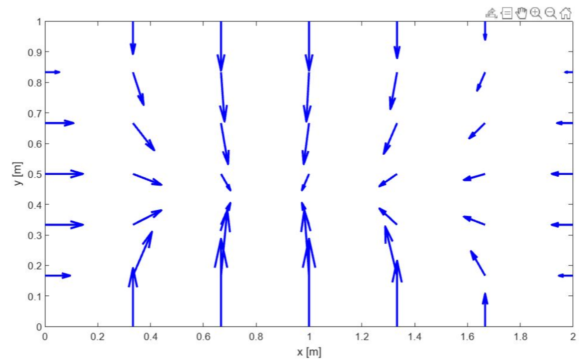

Figure 5: Electric field along a distance

5 Charge-Free Region with Linearly Increasing Potentials on Boundaries

For this question, one should consider a two dimensional charge-free region of space with linearly increasing potentials on its boundaries. This can be accomplished by changing the program from the second problem to suit the electric potentials of zero volts to twenty volts on the left boundary, and ten volts to forty volts on the right boundary. As shown in 6, the electric potential decreases almost linearly along with the distance away from the boundary.

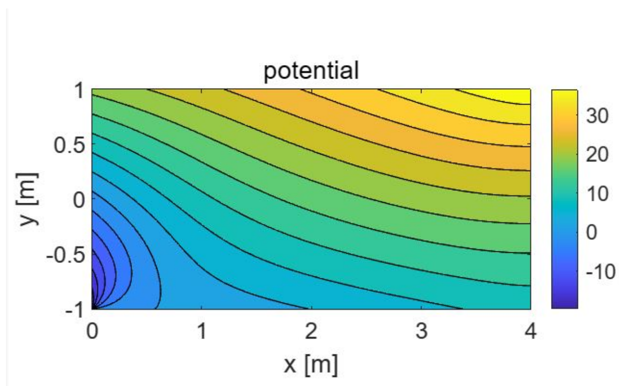

Figure 6: Electric potential across distance in meters