Homework 3 - Aidan Sharpe

1

A 50[kg] homogeneous smooth sphere rests on a 30° incline and against a vertical wall.

Since the sphere is in static equilibrium:

$$\sum \vec{F} = 0$$

$$\therefore \sum \vec{F}_x = \sum \vec{F}_y = 0$$

$$\sum \vec{F}_y = \vec{F}_g + \vec{F}_{A_y}$$

$$\sum \vec{F}_x = \vec{F}_B + \vec{F}_{A_x}$$

Since $\vec{F}_A$ is normal to the surface at 30° to the $-\hat{x}$ direction:

$$\vec{F}_{A_x} = F_A \cos(60°)\hat{x}$$

$$\vec{F}_{A_y} = F_A \sin(60°)\hat{y}$$

Find the force due to gravity:

$$\vec{F}_g = (50)(-9.8)\hat{y} = -490\hat{y}$$

Solve for $\|\vec{F}_A\|$:

$$-490 + F_A \sin(60°) = 0$$

$$\therefore \|\vec{F}_A\| = \frac{490}{\sin(60°)} = 565.8[\text{N}]$$

$$\vec{F}_B + 565.8\cos(60°) = 0$$

$$\therefore \vec{F}_B = -282.9\hat{x}[\text{N}]$$

Since $F_{A_x} = -F_B$ and $F_{A_y} = -F_g$:

$$\vec{F}_A = 282.9\hat{x} + 490\hat{y}[\text{N}]$$

2

A uniform 150[kg], 15[m] long pole is supported by vertical walls spaced 12[m] apart at points A and B. A vertical tension force is applied 5[m] from point A (10[m] from point B). Find the reactions at A and B.

Assumptions: The moment, M_g, acts at the center of mass. The pole pivots around the point where the cable is attached. The forces at points A and B act strictly in the horizontal direction. There is no friction between the pole and the walls.

Find F_A and F_B:

$$\sum \vec{M} = 0 = \vec{M}_g + \vec{M}_A + \vec{M}_B$$

Find the angle that the pole makes:

$$\theta = \arccos\left(\frac{12}{15}\right) = 0.6435$$

Find the about the cable due to gravity, M_g:

$$\vec{M}_g = \vec{r}_g \times \vec{F}_g$$

Since $\vec{r}_g$ is 2.5[m] along the bar from the point of tension:

$$\vec{r}_g = 2.5\cos(\theta)\hat{x} + 2.5\sin(\theta)\hat{y} = 2\hat{x} + 1.5\hat{y}$$

$$\vec{F}_g = mg = (150)(-9.8)\hat{y} = -1470\hat{y}$$

Plug in and evaluate $\vec{M}_g$:

$$\vec{M}_g = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2 & 1.5 & 0 \\ 0 & -1470 & 0 \end{vmatrix} = -2940\hat{z}$$

Find the moment about the cable attachment due to the support from point A, $\vec{M}_A$:

$$\vec{M}_A = \vec{r}_A \times \vec{F}_A$$

Since $\vec{r}_A$ is -5[m] along the pole from the point of tension:

$$\vec{r}_A = -5\cos(\theta)\hat{x} - 5\sin(\theta)\hat{y} = -4\hat{x} - 3\hat{y}$$

Since $\vec{F}_A$ is unknown, but known to only act in the $\hat{x}$ direction:

$$\vec{M}_A = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -4 & -3 & 0 \\ F_A & 0 & 0 \end{vmatrix} = 3\vec{F}_A\hat{z}$$

Find the moment about the cable attachment due to the suport from point B, $\vec{M}_B$:

$$\vec{M}_B = \vec{r}_B \times \vec{F}_B$$

Since $\vec{r}_B$ is 10[m] along the pole from the point of tension:

$$\vec{r}_B = 10\cos(\theta)\hat{x} + 10\sin(\theta)\hat{y} = 8\hat{x} + 6\hat{y}$$

Since $\vec{F}_B$ is unknown, but known to only act in the $\hat{x}$ direction:

$$\vec{M}_B = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 8 & 6 & 0 \\ F_B & 0 & 0 \end{vmatrix} = -6F_B\hat{z}$$

To find the values F_A and F_B:

$$\sum \vec{M} = 0 = \vec{M}_A + \vec{M}_B + \vec{M}_g$$

$$\therefore -2940 + 3F_A - 6F_B = 0$$

$$\sum \vec{F}_x = 0 = \vec{F}_A + \vec{F}_B$$

$$\therefore \vec{F}_A = -\vec{F}_B$$

$$\therefore -2940 = 9\vec{F}_B$$

$$\therefore \vec{F}_B = -326.6\hat{x}$$

$$\therefore \vec{F}_A = 326.6\hat{x}$$

4

If the car accelerates at 2.75[m/s^2] for 3[m] and then maintains speed for 4[m], find the time it takes to travel the entire distance.

$$v^2 = 2a\Delta p = 2(2.75)(3) = 16.5[\text{m/s}]$$

$$v = at$$

$$t_{\text{decline}} = \frac{16.5}{2.75} = 6[\text{s}]$$

$$\Delta p = vt$$

$$t_{\text{coasting}} = \frac{4}{16.5} = 0.242[\text{s}]$$

$$t = t_{\text{decline}} + t_{\text{coasting}} = 6.242[\text{s}]$$

5

A ball is thrown upwards with an initial velocity of 30[m/s] at the edge of a 60[m] high cliff. Find the maximum height above the ground, h, and the total time, t, before the ball hits the ground.

$$0^2 = (30)^2 + 2(-9.8)\Delta h$$

$$\frac{900}{(2)(9.8)} = \Delta h = 45.918[\mathrm{m}]$$

$$h = 60 + \Delta h = 105.918[\mathrm{m}]$$

$$\Delta h_{\mathrm{final}} = 30 + \frac{1}{2}(-9.8)t^2$$

$$-60 = 30 - 4.9t^2$$

$$\therefore t = +\sqrt{\frac{-90}{-4.9}} = 4.2857[\mathrm{s}]$$