Homework Lecture #3

1 Individual Tasks/Problems from Lecture 3:

Given:

Rocket motor equation:	$F = \frac{\dot{W}}{G}\ V_e + (P_e - P_a)\ A_e$
Rocket weight (no propellant):	$300\ kg$
Propellant weight:	$800\ kg$
Acceleration at burnout:	$18\ G$

First things first – the contribution of the pressure differential, $(P_e - P_a)A_e$ is typically very small. Its contribution will be ignored for this problem.

Compute the I_{SP} required to develop a rocket with a burnout velocity of $1000\ m/s$.

Compute the exit velocity $(V_e\ or\ V_j)$ required for this rocket.

Plot the rocket acceleration (in terms of G) and rocket weight as a function of time.

What is the burn time of the rocket?

Finally, plot the speed of the rocket, assuming a flight path angle of 45 degrees assuming no gravity. Plot the speed of the rocket considering gravity.

Homework Lecture #3

2 Aerodynamics Revenge

Overview:

Expand upon the simulation results from lecture 2 with a different type of object. The projectile still does not have a rocket motor, but it is cylindrical in shape and has control surfaces. The following assumptions are true.

- Our earth model is a flat earth model
  - No earth rotation, no curvature, etc.
  - Gravity exists, and it is independent of height (thus $G = -9.81\ m/s^2 \quad \hat{z}$)
- For simplicity, we assume the object has no spin (no moments)
- Physical:
  - Weight = 200 kg
  - Diameter = 0.150 m
- Kinematics:
  - Maximum angle of attack, $AoA_{limit} = 40°$
  - Missile can achieve 30G of acceleration at a height of 15 km when traveling at a speed of 800 m/sec
    - $A_{max} = 30\ G$
    - $h = 15\ km$
    - $v = 800\ m/s$
- Axial drag
  - Mach < 0.5 CA = 0.04
  - Mach > 1.0 CA = 0.02 + 0.06/Mach
  - Otherwise CA = linearly interpolated between values found for Mach 0.5 and Mach 1.0

2.1 Evaluate the how a change in CA affects the projectile

Multiple runs are to be made, each run with a different initial elevation angle (10 through 60 degrees in steps of 5 degrees). We clamp at 60 degrees for the time being such that body orientation angles will not exceed 90 degrees when a normal force is added to our simulation.

To Do:

- For each run, plot the following vs. time (you can put more than more run on a single plot)
  - Object position (y, z total, in units of km)
  - Object velocity (y, z total, in units of m/s)
  - Object acceleration (y, z total, in units of G)
- Plot the final distance and final speed achieved as a function of initial launch angle

Homework Lectures #3

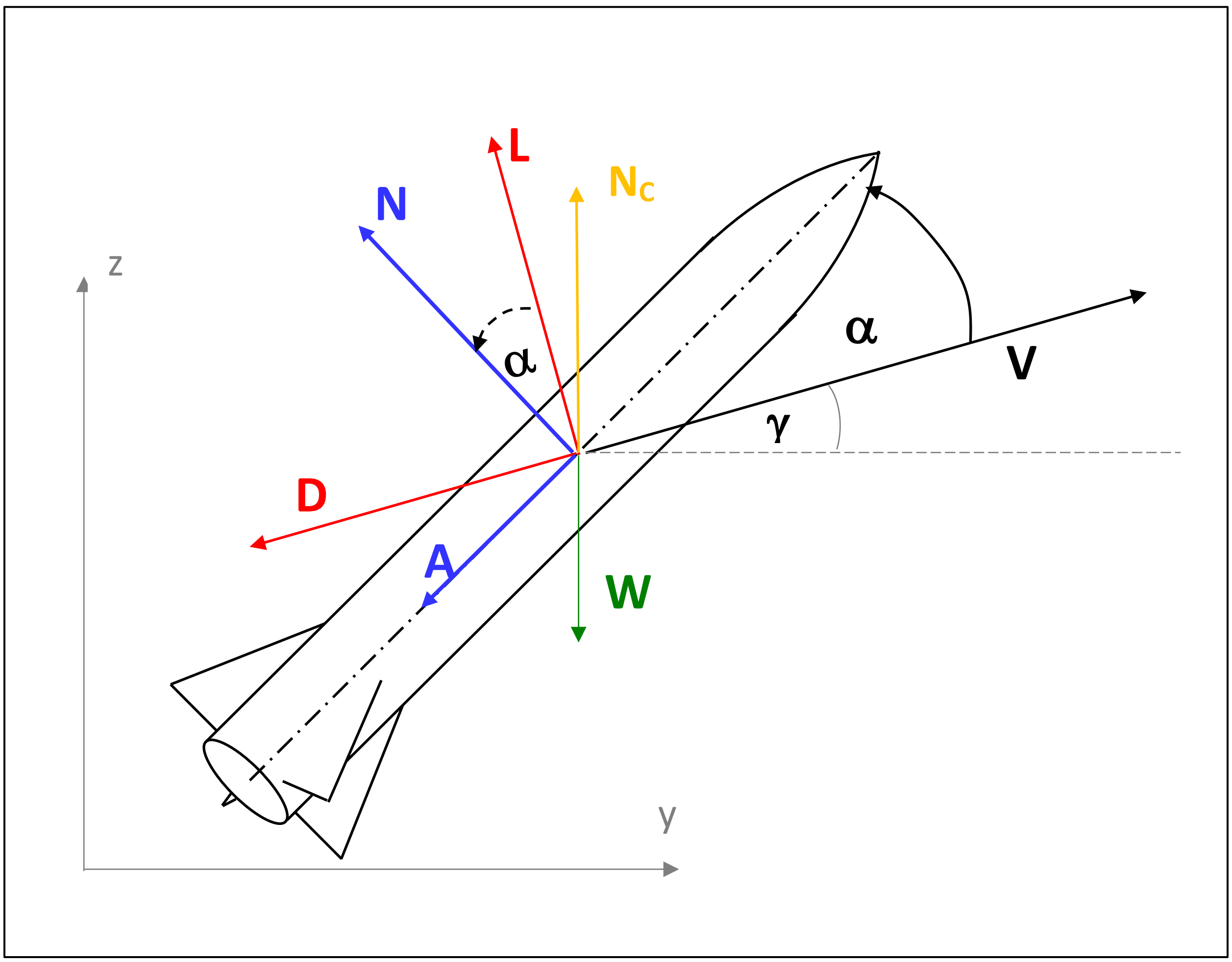
2.2 Adding a normal force to your simulation

Upgrade your model to include the ability to simulate a normal force using the $C_{N_\alpha} = 15$ and the axial force model described above. At this time, the normal force is only meant to compensate for gravity (1 G up – see the yellow/orange N_C in the illustration below).

Set the update period of your integration loop for 0.01 seconds. For each pass through the loop,

- Compute the acceleration normal to the body axis
- Determine the angle of attack required to achieve that acceleration. If the angle of attack is greater than 40 degrees, clamp the acceleration to a value that limit the angle of attack to 40 degrees.
- Compute the drag on the missile (axial and induced)
- Integrate the projectile's position and velocity using the net acceleration computed. The net acceleration should be done in stable coordinate system (y, z)

Hint: You may need to consider both the flight path angle (γ) and the angle of attack (α) to determine the body orientation and the direction of the net acceleration.



See how much further the projectile flies when gravity compensation is included as opposed to the distances in section 2.1.

Does the projectile always achieve an acceleration of “1G up” to compensate gravity? If not, what are some reasons as to why the projectile doesn’t achieve its desired acceleration?