Power Electronics

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Chapter 1

Definition 1.0.1: Power Electronics

Any electronics between the power source and the load.

Definition 1.0.2: Diode

A non-linear, polarized circuit component. Its positive terminal is called the anode, and its negative terminal is called the cathode. If the voltage at the anode is greater the voltage at the cathode, the diode will conduct. Otherwise, the diode behaves as an open circuit.

An ideal diode, when conducting, will have no voltage drop, so it will behave like a direct short. An ideal diode, when not conducting, will allow no current to flow, so it will behave like an open circuit.

For any periodic signal, the average value is given by:

$$V_{\text{avg}} = \frac{1}{T} \int_{0}^{T} x(\omega t) dt$$

Where:

ω is the angular frequency of the signal, $x(\omega t)$

T is the period of the signal, $x(\omega t)$

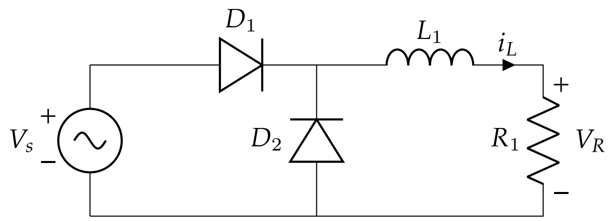
Since a half-wave rectifier only allows current to flow for half of a period, the average voltage out is only a fraction of the input voltage.

1.1 Half-Wave Recifier with Resistive Load and Inductive Filter

Since inductors act like a flywheel for current, when an inductor is placed in series with the half-wave rectifier, the voltage across the load and inductor is not always positive. Adding this inductor, however, allows current to flow through the load for more time during each period. With a properly selected inductance, positive current may be able to flow throught the load for the entire period.

1.2 Freewheeling with Resistive Load and Inductive Filter

Simply by adding a second diode, the inductor will slowly dissipate into the resistor between the positive cycles. This slow dissipations is called *freewheeling*. Increasing the size of the inductor will result in less ripple in the output voltage.



For the first half-cycle ($0 < \omega t < \pi$):
D_1 is on, D_2 is off, and $i_L = i_R$.

$$V_s \sin(\omega t) = L\frac{di_L}{dt} + i_l R$$

$$i_L(t) = \frac{V_s}{Z}\sin(\omega t - \varphi) + Ae^{-Rt/L}$$

For the second half-cyle ($\pi < \omega t < 2\pi$):

$$V_L + V_R = 0$$

$$L\frac{di_L}{dt} + i_L R = 0$$

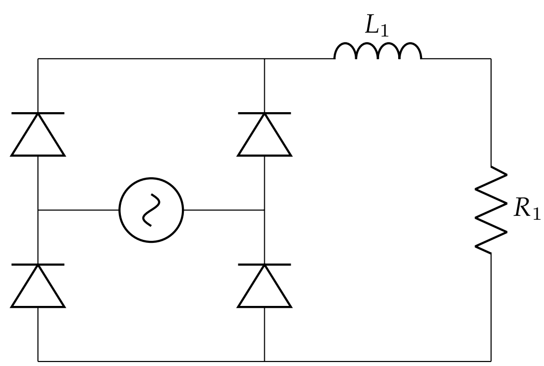
$$i_L(t) = Be^{-R(\omega t - \pi)/\omega L}$$

Since $i_L(\pi)$ must be the same for both the first and the second half-cycles, the two equations must be equal at $t = \pi$.

$$i_L(\pi) = B$$

$$i_L(0^+) = A - \frac{V_s}{Z^2}\omega L$$

1.3 Full Bridge with Resistive Load and Inductive Filter



1.4 Periodic Steady State

The periodic steady state is when a system returns to the same steady state at the end of each cycle.

Example 1.4.1 (Inductor PSS)

$$v_L = L\frac{di_L}{dt}$$

$$\langle v_L \rangle = \frac{1}{T}\int_0^T v_L dt = \frac{1}{T}\int_0^T L\frac{di_L}{dt}dt$$

$$\langle v_L \rangle = \frac{L}{T}\left[i_L(T) - i_L(0)\right] = 0$$

Example 1.4.2 (Capacitor PSS)

$$i_C = C\frac{dv_C}{dt}$$

$$\langle i_C \rangle = \frac{1}{T}\int_0^T i_C dt = \frac{1}{T}\int_0^T C\frac{dv_C}{dt}dt$$

$$\langle i_C \rangle = \frac{C}{T}\left[v_C(T) - v_C(0)\right] = 0$$

1.5 Trigonometric Fourier Series

For any periodic signal with period, $f(t) = f(t + nT), n \in \mathbb{Z}$, where T is the period, a trigonometric series can be constructed to create an identical signal:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos(n\omega t) + b_b \sin(n\omega t))$$

where $\omega = \frac{2\pi}{T}$. The value of a_0 is twice the average value of the signal:

$$a_0 = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)dt.$$

Additionally, the value of a_n and b_n are given by:

$$a_n = \frac{2}{T}\int_{t_0} t_0 + Tf(t)\cos(n\omega t)dt$$

$$b_n = \frac{2}{T}\int_{t_0} t_0 + Tf(t)\sin(n\omega t)dt.$$

The term, $\frac{a_0}{2}$ is the *DC offset* of the signal. While the values of a_n and b_n are defined, since the only difference between sin and cos is a phase shift, the expression for the trigonometric Fourier series can be rewritten as:

$$f(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega t + \varphi_n)$$

where $A_n = \sqrt{a_n^2 + b_n^2}$ and $\varphi_n = -\arctan\left(\frac{b_n}{a_n}\right)$

Chapter 2

Three-Phase Power

2.1 Y and Δ Configurations of AC Voltage Sources

2.1.1 The Y Configuration

The Y configuration is shaped like the letter Y with a neutral connection in the center.

$$V_a = V_a n = V_s \sin(\omega t)$$

V_b lags by 120°.

$$V_b = V_b n = V_s \sin(\omega t - \frac{2\pi}{3})$$

V_c lags by 240°.

$$V_c = V_c n = V_s \sin(\omega t - \frac{4\pi}{3})$$

2.1.2 The Δ Configuration

The Δ configuration is shaped like the letter Δ with no neutral connection.

$$V_a b = V_a - V_b = V_s \left[\sin(\omega t) - \sin(\omega t - \frac{2\pi}{3})\right] = \sqrt{3} V_s \sin(\omega t + \frac{\pi}{6})$$

$$V_b c = V_b - V_c = \sqrt{3} V_s \sin(\omega t - \frac{\pi}{2})$$

$$V_c a = V_c - V_a = \sqrt{3} V_S \sin(\omega t + \frac{5\pi}{6})$$

2.1.3 Y Configuration Power

If a Y configuration of AC sources is connected to a Y configuration of resistors, the power through to each load resistor is:

$$P_a = \frac{V_s^2}{2R_a}\left[1 - \cos(2\omega t)\right]$$

$$P_b = \frac{V_s^2}{2R_a}\left[1 - \cos(2\omega t - \frac{4\pi}{3})\right]$$

$$P_c = \frac{V_s^2}{2R_c}\left[1 - \cos(2\omega t + \frac{4\pi}{3})\right]$$

The total power transfer is:

$$P = 3\frac{V_s^2}{2R}$$

where $R = R_a = R_b = R_c$.

One benefit of using three-phase is constant power from the source. Another advantage of using three-phase power is that any harmonics divisible by either 2 or 3 cancel out. The first harmonic after the fundamental is the 5^{th} harmonic, followed by the 7^{th}, 11^{th}, and 13^{th}. Harmonics are non-zero for $6n \pm 1, n \in \mathbb{Z}$.

Chapter 3

Silicon Controlled Rectifier (SCR)

The SCR will be off even when $V_{AC} > 0$ until a pulse is applied to the gate. Once the pulse is applied, the device will stay on as long as $i_F > 0$.

Note:

The voltage across the SCR can go negative as long as the current remains positive.