Simulating Simple Electrostatic Capacitors

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***Abstract*—The primary goal of this lab was to understand the electric fields between differently charged conductors. Since the definition of a capacitor is two separated conductors of differing charge, capacitance was also predicted and measured. Along with a simple parallel plate model, coaxial conductors were also analyzed and found to change the characteristics of the electric field.**

I. Introduction & Methods

The idea of a charged object in a vacuum can be quite helpful when developing simple models and fundamental understandings. Unfortunately, this approach can only be taken so far before a system becomes *oversimplified*. To better understand these complex interactions, more complex models must be made.

Parallel plate capacitor with square plates of side length 0.5[mm], and spacing 0.1[mm]. The bottom plate was defined as 0[V], and the top plate was given a charge, 5×10^{-6}[C]. Since the measurements are being taken with air between the plates, and air has a relative permittivity of 1.0006, for air, $\varepsilon = 8.859 \times 10^{-12}$. The capacitance was calculated by plugging the known values for area and distance into equation 1. Doing so yields the theoretical capacitance of 2.215×10^{-14} [F].

$$C = \frac{\varepsilon A}{d} \tag{1}$$

Where:
C is capacitance
A is the inside area of the plates
d is the distance between the plates
ε is the permittivity of the material between the plates.

II. Results & Analysis

A. Air as a Dielectric

After setting up the capacitor, analysis began. First, as seen in figure 1 and figure 2, the electric field between the plates was analyzed at the edge of the plates and through the center of the area between the plates. It is immediately apparent that changing the location of measurement inside the region between the plates does not affect the results of the measurement. For this reason, we can determine that the electric field strength between the plates is constant. Additionally, the strength of the electric field outside the region between the plates drops off quickly and is very small above and below the plates.

Since one of the plates was charged and the other was set as a reference ground, there exists some voltage between the two plates. This voltage can be found using equation 2. To calculate this voltage, the electric field strength was measured along a straight line from the center of one plate to the center of the other. The resulting voltage was 196[MV].

$$V = \int \vec{E} \cdot d\vec{l} \tag{2}$$

To find the experimental capacitance, equation **??**. Using the predetermined charge of 5×10^{-6}[C] and the tool evaluated

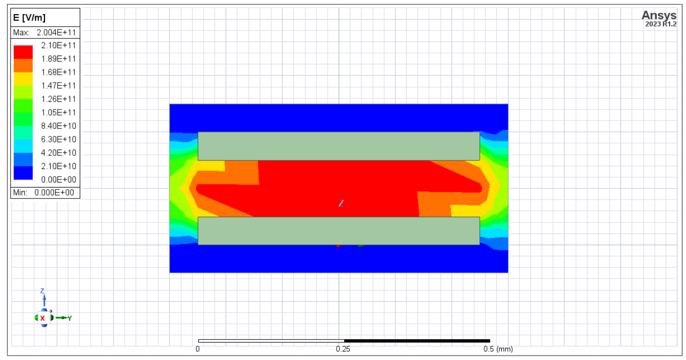

Fig. 1. Measuring electric field strength at the edge

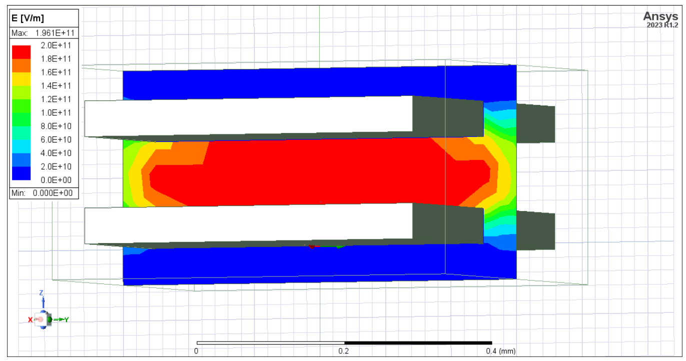

Fig. 2. Measuring electric field strength in the center of the capacitor

line integral for voltage, the experimental capacitance was determined to be 2.549×10^{-14}[F]. The assumption of a uniform electric field between the plates is likely the source of the error. Since the plates have thickness, modelling them as plane charges will cause some discrepancy.

B. Changing the Dielectric

The region surrounding the plates was air in this context, however, changing the material between the plates will change the capacitance. However, changing this material into a conductor will short the plates and effectively ruin the capacitance. For this reason, the material was chosen to be diamond for its insulating and dielectric strengths. Diamond has a relative permittivity of 16.5. Therefore, it should be expected that the capacitance be multiplied by the same factor. Additionally, since capacitance is increasing and charge is remaining constant, the voltage between the plates should reduce by the same factor as well. Sure enough, the evaluated voltage between the plates is 11.86[MV], a factor of 16.53 smaller.

Similarly, since voltage is related to electric field strength by distance, and the distance is constant, the electric field strength should also decrease. This is verified by the test results shown in figure 3. With diamond as a dielectric, the maximum field strength was 1.192×10^{11}[V/m] compared to 2.004×18^{11}[V/m] with air as a dielectric.

C. Changing the Form of the Conductors

Finally, the dielectric was reset to air, and for this test, the parallel plates were replaced with concentric conductors in the shape of a coaxial cable. Since the surface area of the inner conductor is smaller than the surface area of the

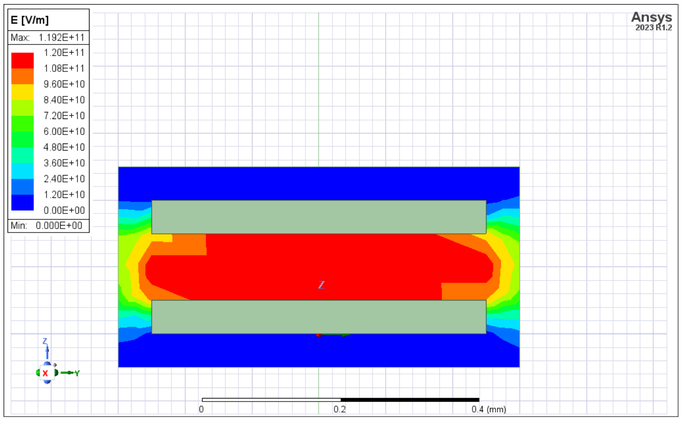

Fig. 3. Using diamond as a dielectric

outer conductor, the electric field cannot be uniform between them. As seen in figure 4, the electric field is stronger around the inner conductor, and drops off until the surface of the outer conductor. It is also apparent that there is no electric field beyond the outer conductor, so it has some shielding effects. Additionally, equation 1 only applies to parallel plate capacitors, the capacitance of the coaxial conductors was evaluated experimentally.

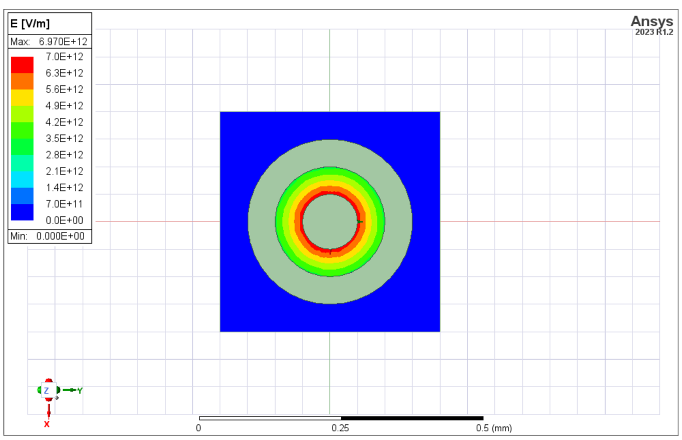

Fig. 4. Electric field between coaxial conductors

The charge of the inner conductor was set to the same 5×10^{-6}[C] as before, and the outer conductor was defined as a reference ground. The radius of the inner conductor was 0.05[mm], the inner radius of the outer conductor was 0.1[mm], and the outer radius of the outer conductor was 0.15[mm], leaving a 0.05[mm] gap between the conductors. The line integral evaluated voltage was 248[MV] between the conductors. Again using equation 2, the capacitance was calculated to be 2.016×10^{-14}[F].

III. Conclusions

Using a parallel plate capacitor is quite limiting. Since capacitance is larger with high surface area, getting larger capacitance requires very large plates. Beyond the range of picofarads, other capacitor form factors are preferred for their compactness.

Additionally, parallel plate capacitors, are superb for creating a uniform electric field. However, they are also create electromagnetic disturbances in the surrounding region. This can be reduced with shielding, but adding this shielding will affect the quality and uniformity of the internal electric field. This behavior is exemplified by the coaxial cable. There are no disturbances surrounding the central conductor, but this comes at the cost of lost uniformity.

Overall, the primary takeaway is that the form factor makes a large contribution to a capacitors behavior. There are trade-offs to picking one design over another, so application must be considered.