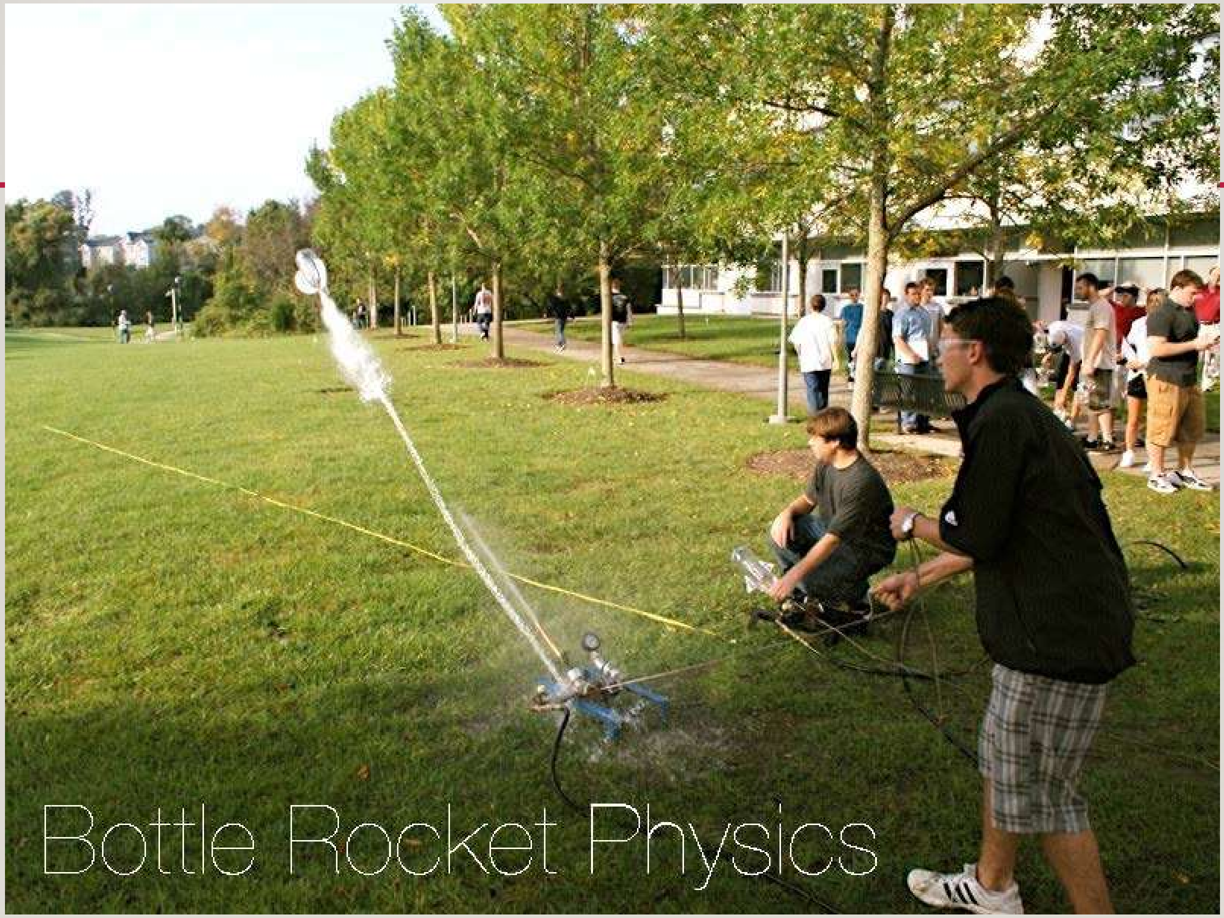
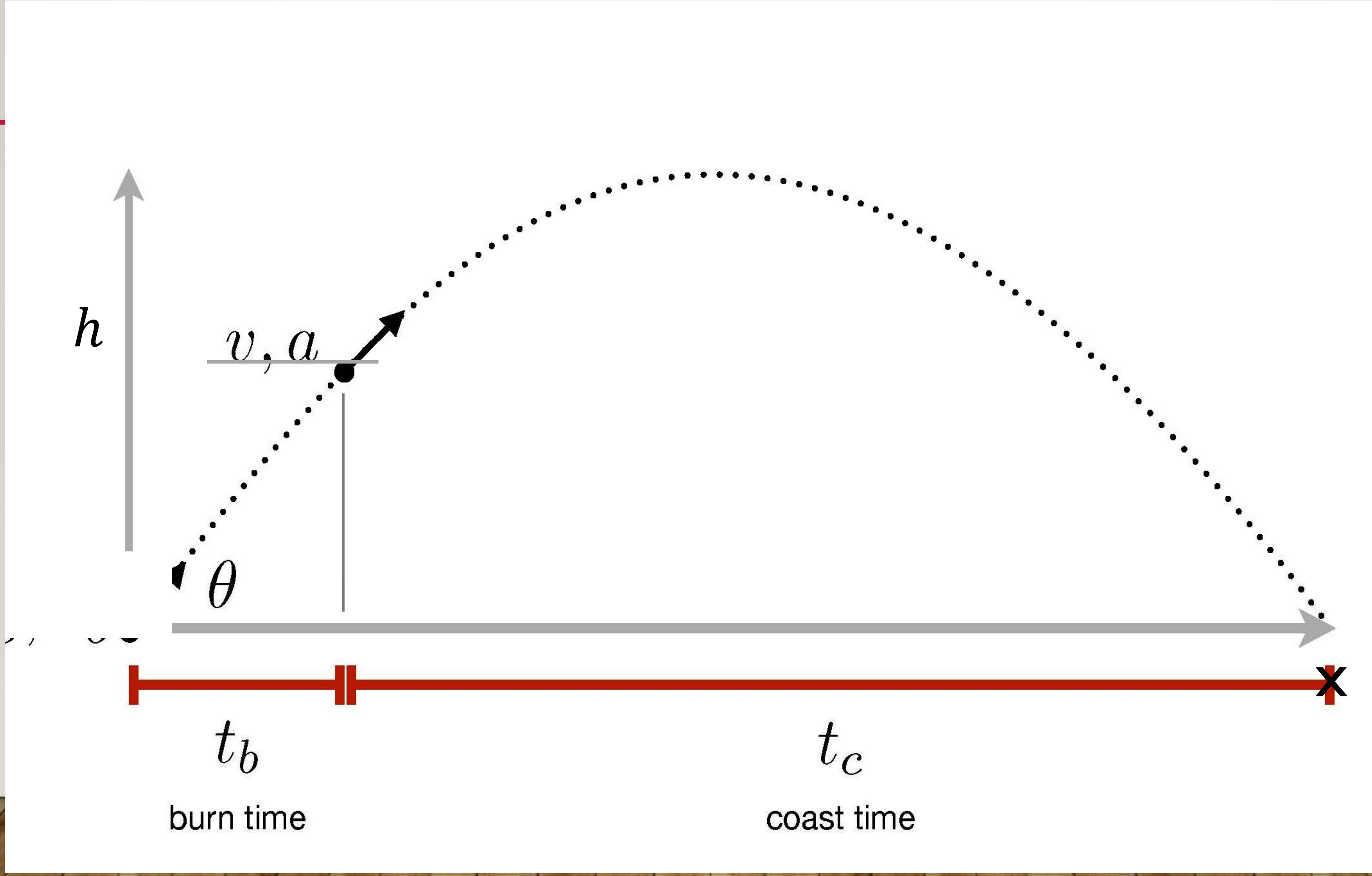
Sophomore Engineering Clinic I – Fall 2019

SOPHOMORE ENGINEERING CLINIC (SEC) I

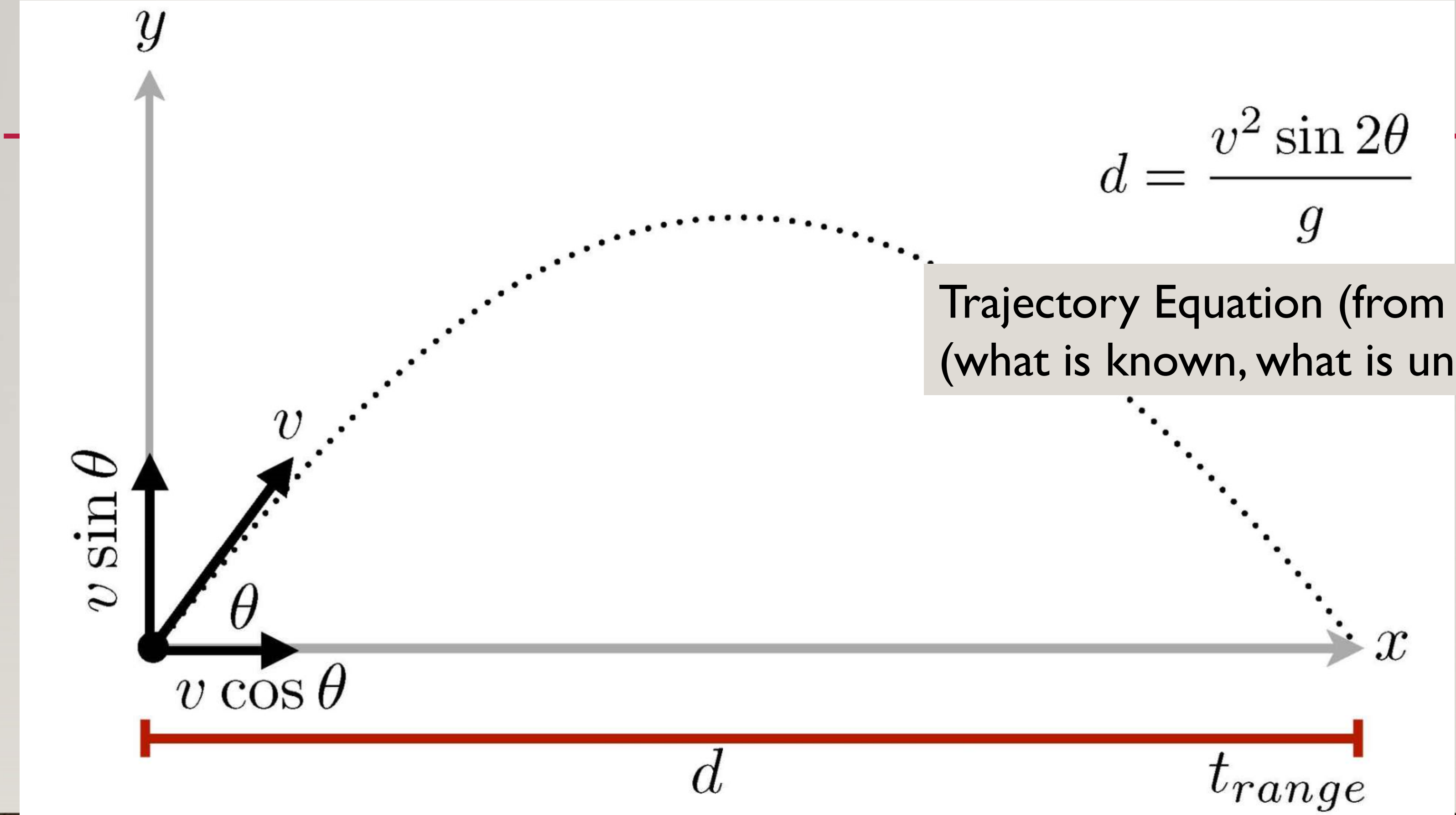
WEEK 3 – BOTTLE ROCKET PHYSICS


Bottle Rocket Physics

Bottle Rocket Flight has Two Stages



How Do We Model Range (d)?



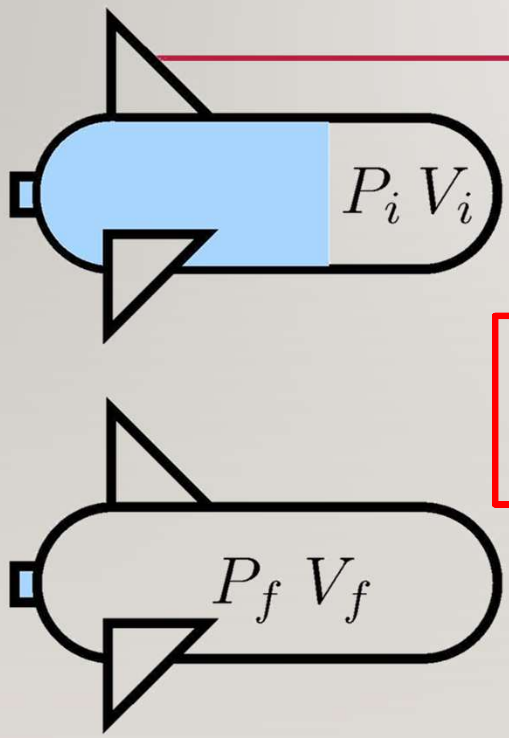
$$d = \frac{v^2 \sin 2\theta}{g}$$

Trajectory Equation (from physics?)
(what is known, what is unknown?)

For θ=45° , rocket is down range a distance (d) at the end of burn

BOTTLE ROCKET MODEL

$$W = \int dW = \int_{V_i}^{V_f} P * dV$$



$$PV = nRT$$ (Ideal gas law) from chemistry?

$$P = \frac{nRT}{V}$$

$$W = \int_{V_i}^{V_f} \frac{nRT}{V} dV = nRT \int_{V_i}^{V_f} \frac{1}{V} dV = nRT * \ln\left(\frac{V_f}{V_i}\right) = P_i V_i \ln\left(\frac{V_f}{V_i}\right)$$

Integration (from calc?)

$$, where \; V_f = V_b$$

$$and \; V_i = V_b - V_w$$

WORK-ENERGY MODEL, NOW SOLVE FOR "VELOCITY = F(W)"

- Assume all rocket energy is either Potential Energy or Kinetic Energy.
- By conservation of energy we can write: $\frac{1}{2} mv^2 + mgh = W$
- Solving for velocity, v:

$$v = \sqrt{2\left(\frac{W}{m} - gh\right)}$$

In the model we will assume:

- average mass, m during burn stage is half the initial water mass + bottle mass + clay mass;
- use calculated work, w from the burn stage (remember this was a function of P&V);
- to solve for initial velocity (where height is zero, h=0).

WORK-ENERGY MODEL REVIEW – SOLVE FOR "D"

$$W = P_i V_i \ln\left(\frac{V_f}{V_i}\right)$$

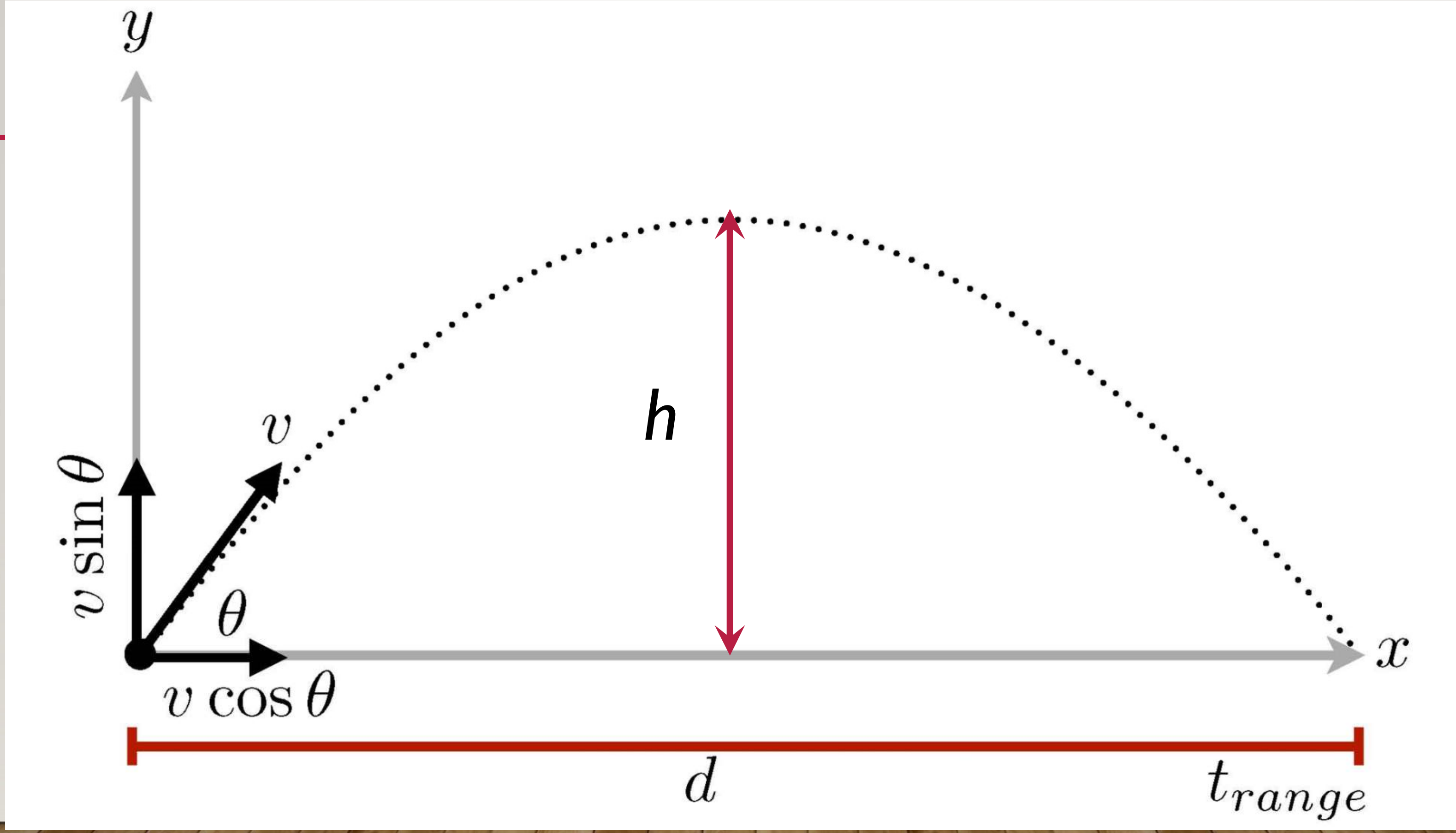
$$m = \frac{\rho_w V_w}{2} + m_{rocket} + m_{clay}$$

$$v = \sqrt{2\left(\frac{W}{m} - gh\right)}$$

$$d = \frac{v^2 \sin 2\theta}{g}$$

W = Work
P_i = Initial pressure in the bottle
V_i = Initial volume of air in the bottle
V_f = Final volume of air in the bottle
m = Avg mass of rocket during burn
ρ = Density of water
V_w = Volume of water in the bottle
v = Velocity of rocket
g = Gravity
h = Height
Θ = Launch angle, 45°
d = Distance

How Do We Model Height (h)?

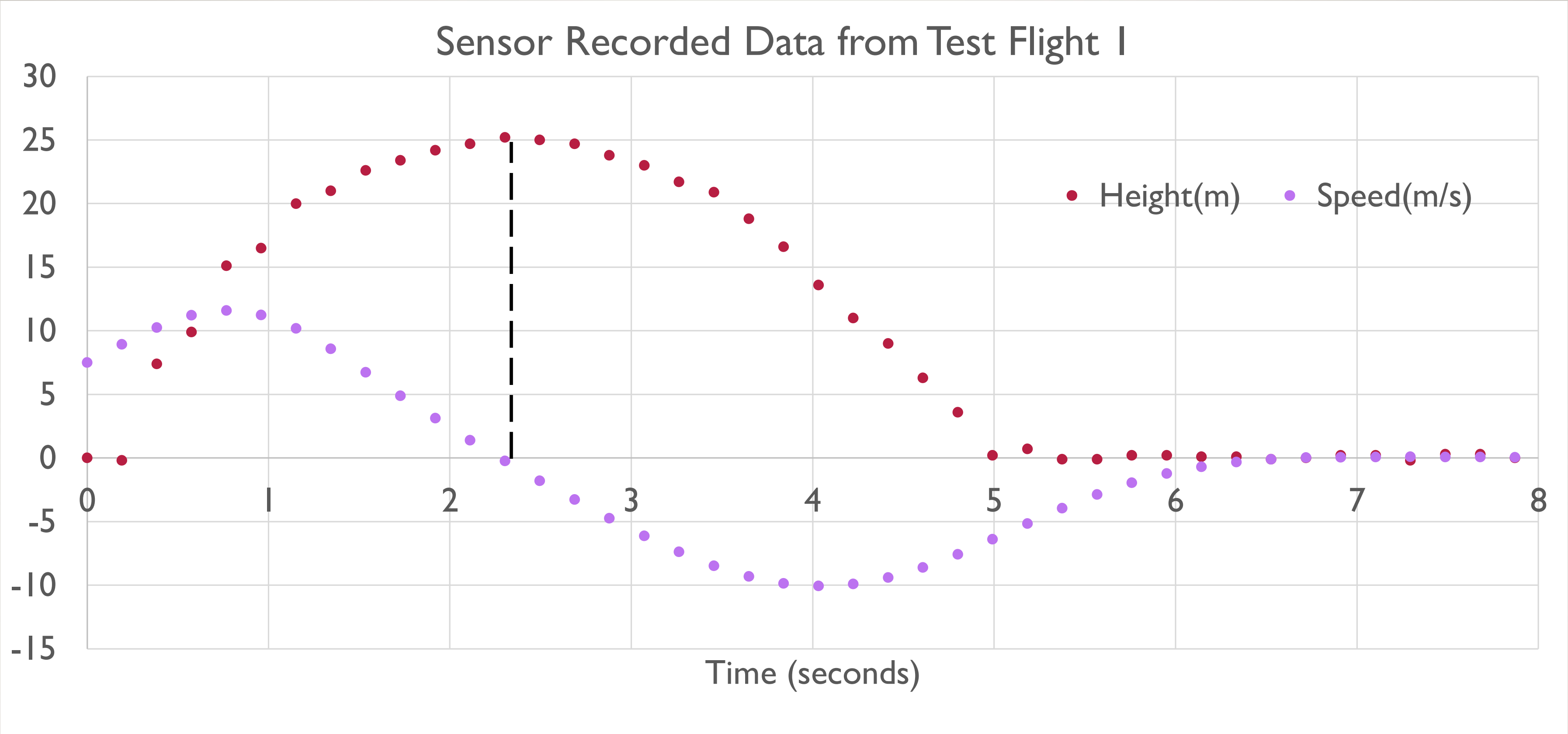


WORK-ENERGY MODEL, NOW SOLVE FOR "HEIGHT = F(W)"

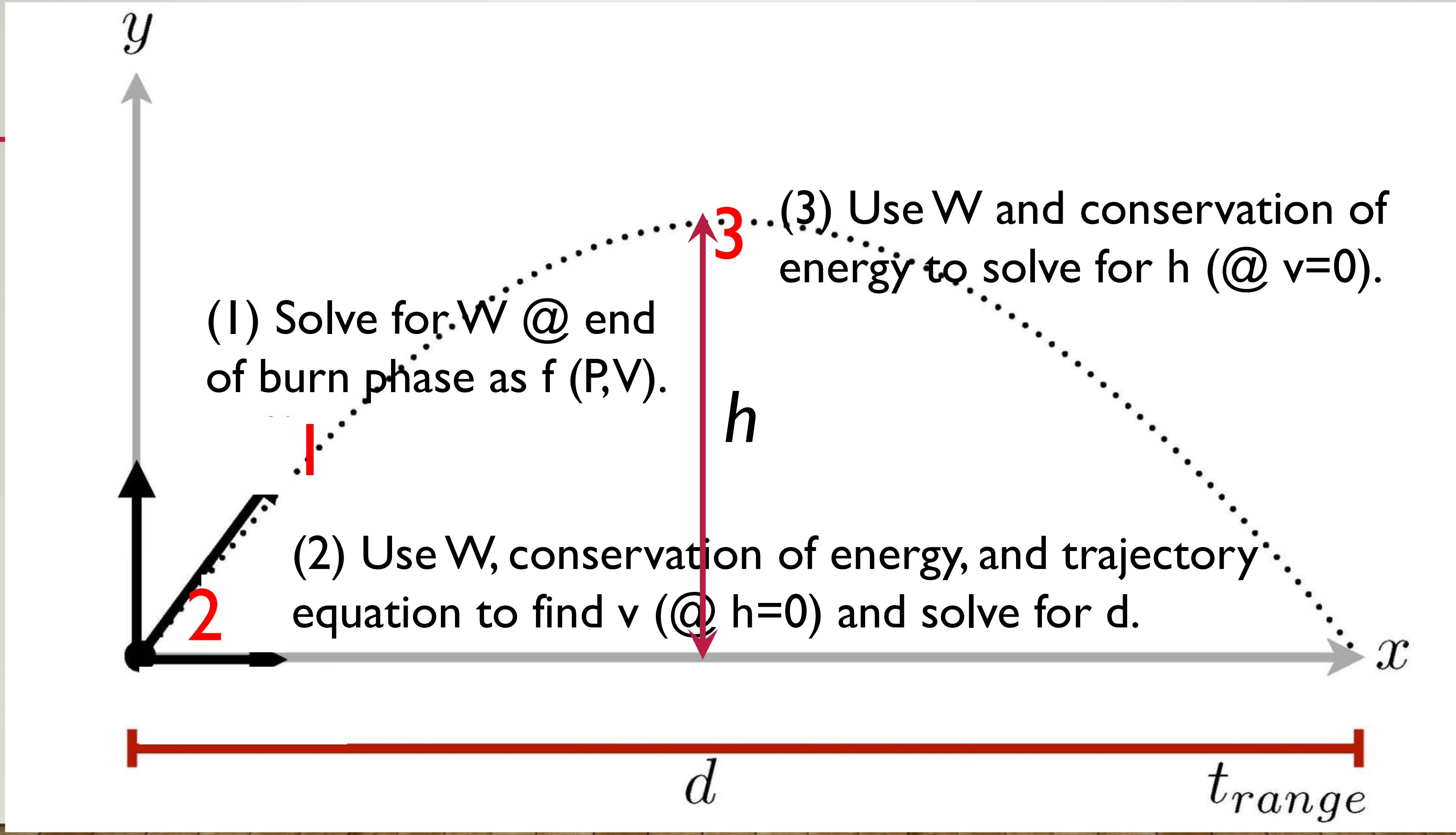
- Assume all rocket energy is either Potential Energy or Kinetic Energy.
- By conservation of energy we can write: $\frac{1}{2}\ mv^2 + mgh = W$
- In the model we will assume:
  - average mass, m during burn stage is half the initial water mass + bottle mass + clay mass;
  - use calculated work, w from the burn stage (remember this was a function of P&V);
  - and solve for maximum height, h, where velocity equals zero.
- Rearrange equation to solve for height, h:

$$h = W/mg$$

EXAMPLE OF SENSOR DATA FROM A FLIGHT TEST…



WORK-ENERGY MODEL REVIEW – SOLVE FOR “D” AND “H”



EXPECTATIONS IN THE BOTTLE ROCKET REPORT

MODELING IN YOUR BOTTLE ROCKET REPORT

- You are expected to:
  - Compare your experimental design to the mathematical model
    - Did you obtain similar results for "d"? If not, why?
  - Compare your sensor data to the mathematical model
    - Did you obtain similar results for "h"? If not, why?
  - Do your results make sense based on the laws of physics?
    - What are some variables to consider? (wind, temperature, lateral component of distance, what else??)