Missile Aerodynamics

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Overview

- A brief discussion of aerodynamics is needed in order to appreciate the complexity of weapon system design
  - We're not planning to build a missile, but…
  - We need to understand basic aerodynamic principles in order to design a weapon system
- The aerodynamic building blocks to be discussed include
  - Basic aerodynamic principles (forces and moments)
  - Missile body type characteristics
  - Projectile stability
- These building blocks will be referenced in other lectures as we discuss numerous topics
  - Guidance law development
  - Weapon system design
  - Track processing (track filtering)
- First, some basic terms used in aerodynamics must be discussed

Mach

Important Terms in Missile Engineering

- Speed can be measured using the speed of sound as a base scale
- This measurement of speed is referred to as Mach (M)

$$M = \frac{V_{Missile}}{V_{Sound}}$$

- Mach number represents the ratio of ordered energy to random energy
- All aerodynamic coefficients are affected by the Mach number.
  - Variations with Mach number become more apparent as M > 1
- Sound travels at 1116 ft/sec at sea level
  - The speed of sound varies as a function of ambient temperature

$$V_{Sound} \cong 49\sqrt{T_{amb}}$$

* **T_{amb}** is temperature in Rankine (518.7° at sea level)

Dynamic Pressure (Q)

Important Terms in Missile Engineering

- Q is defined as

$$Q = \frac{\rho}{2} V^2 \cong \boxed{(0.7)(P)\, M^2} \longleftrightarrow$$

This is the more common definition when M > 0.5

- Where

ρ is the ambient density (slugs/ft^3)	(kg/m^3)
V is the missile velocity (ft/sec)	(m/sec)
P is the ambient pressure (lbs/ft^2)	(Pa = N/m^2)
M is the Mach number	
0.7 is approximately one half the specific heat ratio of air	

- Q is a function of both speed and altitude

The Ability of the Missile to Maneuver is Directly Related to the Dynamic Pressure of the Missile

What – a – sonic?

Important Terms in Missile Engineering

- Steamlines define how air flows past the body as the body cuts through the atmosphere
- Streamlines differ as the speed in which air flows past the body increases
- Four regions of missile speed
  - Subsonic (Mach < 1.0)
  - Transonic (Mach ~ 1.0)
  - Supersonic (1.0 < Mach < 5.0)
  - Hypersonic (Mach > 5.0)
- Air flow patterns over specific geometric shapes (correlating to specific missile parts) are dependent upon the regions listed above

Pressure Waves

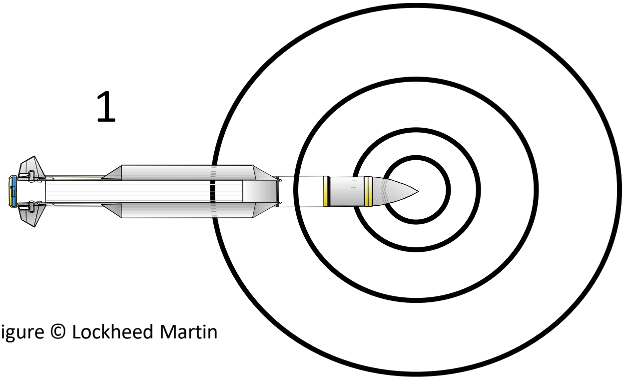
Important Terms in Missile Engineering

- ☐ Pressure waves are the result of molecules of air being pushed together due to a body forcing its way through the atmosphere
- ☐ Pressure waves travel at the speed of sound (Mach 1)
- ☐ They travel in all directions from the disturbance
  - ➢ Similar to waves in a pool after throwing a rock in the water
  - ➢ As the speed of the flying body increases, the shape of the pressure circles change

The Next Slide Illustrates Pressure Waves for Various Missile Speeds

Pressure Waves

1



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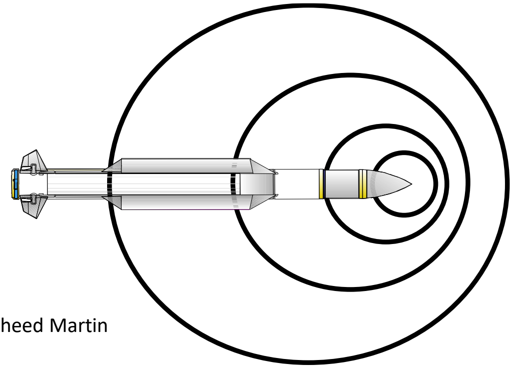
Pressure waves for a very slow moving body

2



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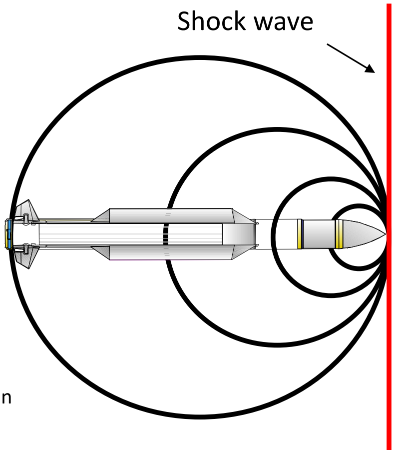
Pressure waves for a body traveling at subsonic speeds

3



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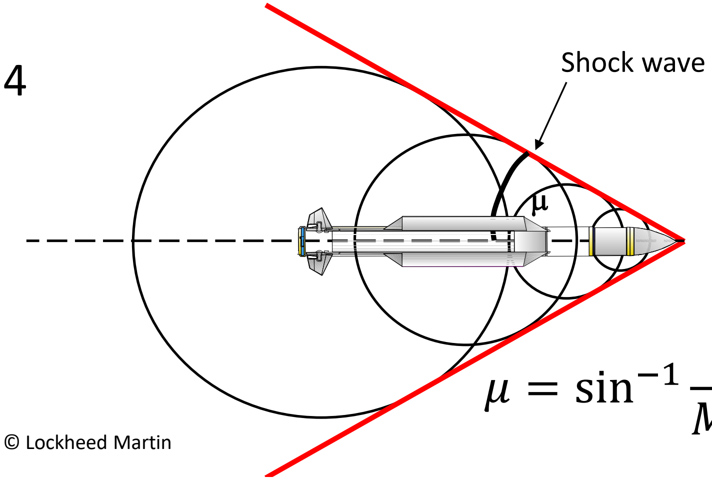
A shock wave is created where the air flow past the body is equal to the speed of the pressure wave (Mach 1)

4



$$\mu = \sin^{-1}\frac{1}{Mach}$$

Figure © Lockheed Martin

The angle, μ, at which a supersonic shockwave is formed can be calculated

Radome/Seeker

Important Terms in Missile Engineering

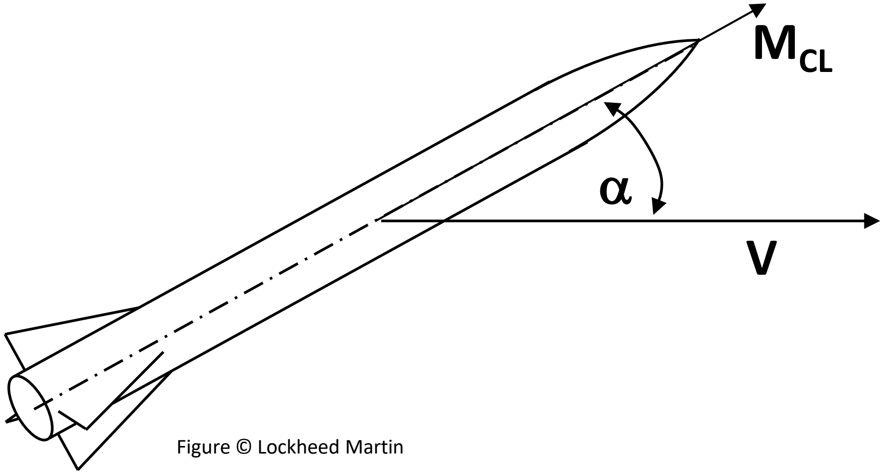
- The missile seeker is often found within the nose cone of the missile and is the device used to detect the missile's intended target
- The radome is the comprised of extremely hard material that allows either IR of RF waves to pass from the environment to the missile seeker
  - The shape of the radome greatly influences the missile's aerodynamic performance as well as its ability to track a target effectively
- The most common seeker types are described below

Seeker Type	Description
Active	Radar located in the nose of the missile that transmits and receives RF signals
Semi-Active	Receiver located in the nose of the missile that receives RF signals reflected off the target which had originated from a source not located on the missile
Passive	Receiver receives signals emanating from the target but the source of the signal was not provided by the combat system
Infrared (IR)	Tracks the target using infrared (heat) imaging

Angle of Attack

Important Terms in Missile Engineering

- ☐ Angle of attack (α) is the angle between the missile body center line (M_{CL}) and the velocity vector of the missile.
  - Also referred to as the angle between the missile body axis and the wind axis.
- ☐ Angle of attack
  - Increases drag on the missile (more missile body is exposed)
  - Angle of attack is required for a missile to change direction



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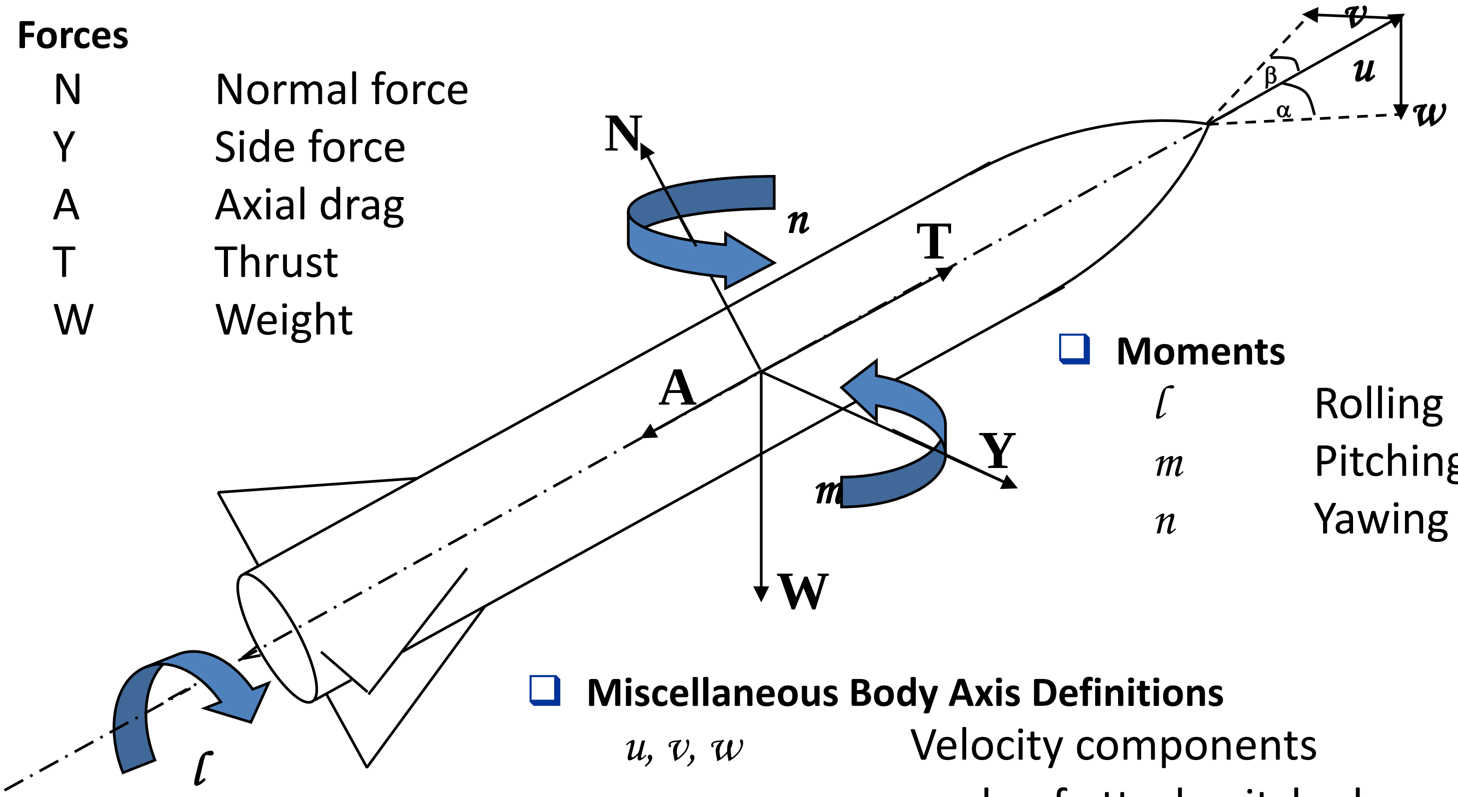
Forces and Moments

- ☐ Forces and Moments are defined in a body axis reference system
  - N – normal force (force perpendicular to the body axis)
  - Y – side force (force perpendicular to the body axis)
  - A – axial force (force along body axis, thrust and drag)
  - m – pitching moment about the lateral axis
  - n – yawing moment about the normal axis
  - l – rolling moment about the center line axis

There is a Great Picture on the Next Page to Illustrate This…

Forces and Moments



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❑ **Forces**

N	Normal force
Y	Side force
A	Axial drag
T	Thrust
W	Weight

❑ **Moments**

ℓ	Rolling moment
m	Pitching moment
n	Yawing moment

❑ **Miscellaneous Body Axis Definitions**

u, v, w	Velocity components
α	angle of attack, pitch plane
β	angle of sideslip, yaw plane

Force Coefficients

- Force coefficients:

$$C_N = \frac{N}{Q\,S_{Ref}} \qquad C_Y = \frac{Y}{Q\,S_{Ref}} \qquad C_A = \frac{A}{Q\,S_{Ref}}$$

- Where:

  S_{Ref} is the reference area

- For missiles, S_{Ref} is usually considered to be the maximum cross-sectional area
  - Compute the reference area using the diameter of the missile (cross sectional area)
- For airplanes, S_{Ref} is defined by the area of the wing

Moment Coefficients

- ☐ Moment coefficients:

$$C_m = \frac{m}{Q\, S_{Ref}\, L_{Ref}} \quad C_n = \frac{n}{Q\, S_{Ref} L_{Ref}} \quad C_l = \frac{l}{Q\, S_{Ref}\, L_{Ref}}$$

- ☐ S_{Ref} is the reference area (max. body cross sectional area)
- ☐ L_{Ref}
  - For missiles, L_{Ref} is a reference length, or body diameter (more on this later)
  - For airplanes, L_{Ref} is related to the wing chord
- ☐ Moment coefficients are taken about a referenced center of gravity
  - Center of gravity changes as fuel/propellant is burned off
  - i.e. the interceptor rotates about its center of gravity due to a non-zero moment

Force and Moment Coefficients

Generic Properties

- Coefficients vary as a function of
    - Mach (M)
    - Angle of attack (α)
    - Sideslip (β)
    - Roll (ϕ)
- Coefficients are determined by theoretical design as well as wind tunnel test analysis
- Coefficients are referred to as proportionality factors
- Scalability
    - Makes wind tunnel work with models very effective
    - Aerodynamic forces scale by L^2 for similar configurations
    - Aerodynamic moments scale by L^3 for similar configurations

Force and Moment Coefficients

Specific Properties

- C_N and C_m are proportional to the angle of attack (α)
  - Influenced by β
- C_Y and C_n are proportional to the side-slip angle (β)
  - Influenced by α
- C_A is independent of both α and β.
- C_l is dependent upon both α and β

Force and Moment Coefficients

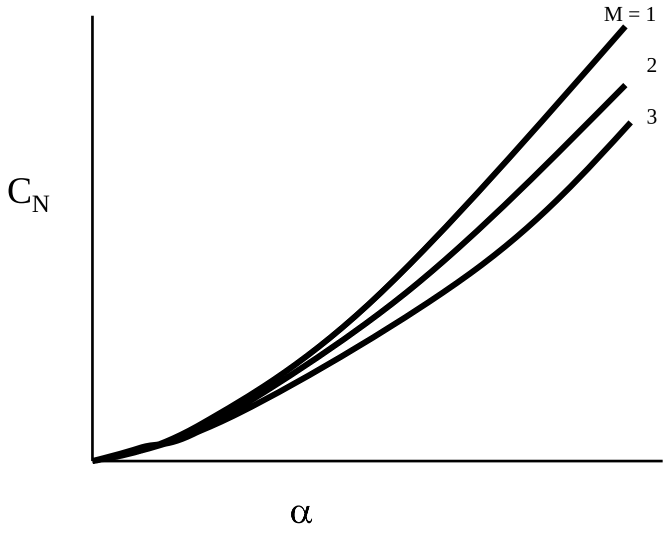
Specific Properties

- Axial drag (A) is far more sensitive to Mach than the normal force (N) or side force (Y)
- Normal force is nearly independent of Mach
  - Provided the wings are slender and pointed
  - Low aspect ratio (delta shape)
- Coefficients associated with the missile's rotational rates
  - Very complex
  - Burdensome to compute
  - They are not covered in this discussion

Force and Moment Coefficients

C_N (or C_Y)

- A typical C_N vs. α plot at various Mach numbers would look like this:



$$C_N \cong C_{N_\alpha}\,\alpha$$

- For each Mach number, C_N as a function of α is nearly linear for small α
- This allows us to write the following:

$$C_N = \left(\frac{\partial\, C_N}{\partial\, \alpha}\right)_M \alpha$$

- This means that each C_N is linear as a function of Mach
- One can see that to get a good first order approximation, one could assume an average Mach resulting in

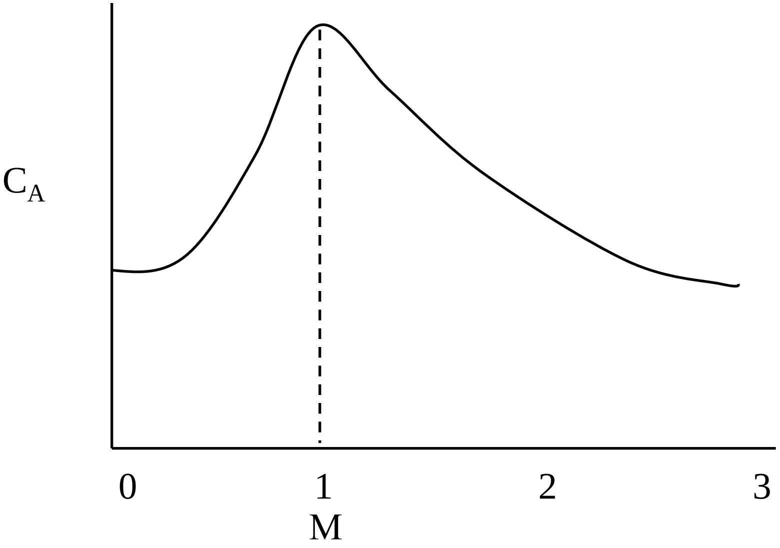
$$C_{N_\alpha} = \left(\frac{\partial\, C_N}{\partial\, \alpha}\right)_{\bar{M}}$$

C_{N_α} Is Considered a Constant!

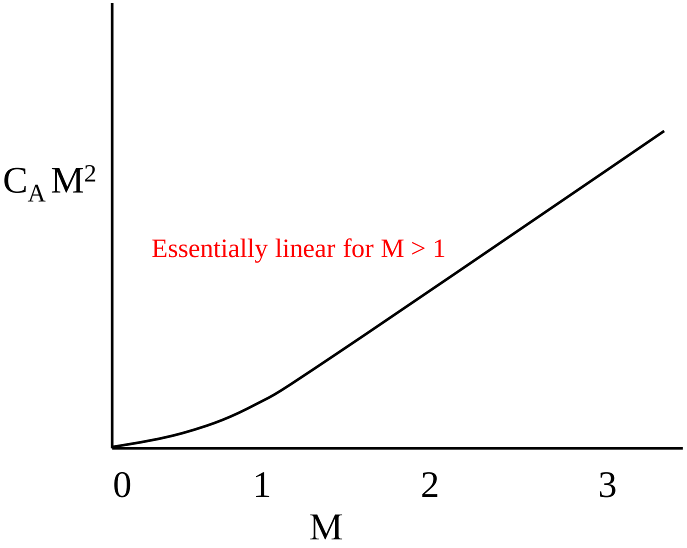
Force and Moment Coefficients

C_A

- ❑ A typical C_A vs. Mach plot would look like this:
- ❑ Since C_A is not very easy to describe as a function of M, we often express it in this manner



$$C_A = Yuck!$$



$$C_A M^2 \cong C_1 M + C_2 M^2 \text{ or } C_A \cong \frac{C_1}{M} + C_2$$

Induced Drag

- The induced drag (C_{D_I}) is the component of lift that resists missile motion
- It is the penalty one must pay to generate lift force on the missile
- The maneuvering efficiency (per G) is a function of induced drag

$$C_{D_I} = C_N\alpha = \left(C_{N_\alpha}\right)\alpha^2$$

- One can represent maneuver G’s as

$$n_z = \frac{N}{W} = \left(C_{N_\alpha}\right)\frac{\alpha\, Q\, S_{Ref}}{W}$$

- Therefore

$$\frac{n_z}{\alpha} = \left(C_{N_\alpha}\right)\frac{Q\, S_{ref}}{W}$$

Induced Drag vs Acceleration Squared

- ☐ Induced drag is the penalty paid for a change of direction by the missile
- ☐ It can be expressed as a function of the square of the acceleration.

$$C_{D_I} = \frac{n_z^2 \, W^2}{(C_{N\alpha}) \, Q^2 \, S_{ref}^2}$$

- ☐ Note which factors contribute to high induced drag, and which contribute to low induced drag
- ☐ The objective of a guidance law is for the missile to hit the target with the greatest amount of kinetic energy (velocity) possible
  - ➢ When developing a guidance law, the goal is to minimize the integral of the commanded acceleration over the time of flight as it should also reduce the induced drag
  - ➢ i.e.

$$I = \int_0^T n_z^2 \; dt$$

Lift and Drag Forces

- ❑ Normal and axial forces translate into lift and drag forces
  - ➢ Lift force is perpendicular to the velocity vector
  - ➢ Drag force is along the velocity vector
  - ➢ Sometimes it is desired to have the forces defined by the orientation of the missile velocity vector rather than the missile center line
- ❑ *There is no lift without drag*
  - ➢ It is a basic property of aerodynamics

It is Desirable to Have a High Lift-To-Drag Ratio

Comparing Forces

Lift and Drag vs Normal and Axial

- **Forces**

N	Normal force
A	Axial drag
L	Lift
D	Drag

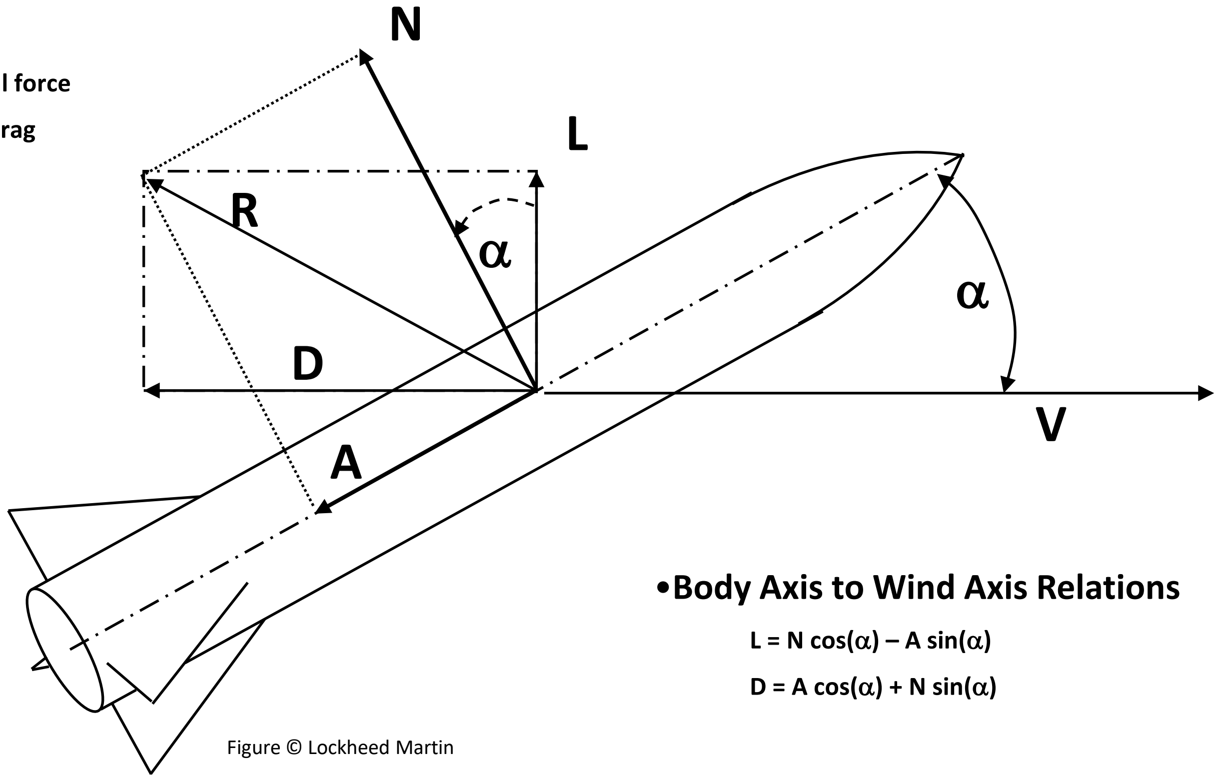


Figure © Lockheed Martin

- **Body Axis to Wind Axis Relations**

  $L = N\cos(\alpha) - A\sin(\alpha)$

  $D = A\cos(\alpha) + N\sin(\alpha)$

- **Miscellaneous Definitions**

V	velocity vector
α	angle of attack, pitch plane

Computing Lift and Drag

- Lift and drag forces describe the forces acting on the missile in a plane defined by the missile velocity vector and the directions orthogonal to it

- The Normal and Axial forces can be defined as Lift and Drag terms as follows:
  - $L = N\cos(\alpha) - a\sin(\alpha)$
  - $D = A\cos(\alpha) + N\sin(\alpha)$

- We can simplify this to
  - $L \cong N$
  - $D \cong A + N\,\alpha$

- Lift and Drag can be normalized in the same manner as the Normal and Axial forces
  - $C_L \cong C_N$
  - $C_D \cong \underset{\uparrow}{C_A} + \boxed{C_N\,\alpha}$

Zero-lift drag

Induced drag, C_{D_I}

Static Margin

- ☐ The missile must be designed for stability during flight
  - Forces and moments acting upon the interceptor can easily result in an unstable flying configuration
  - One of the key concepts in understanding stable flight is static margin
- ☐ Static Margin is defined as the distance the center of gravity is aft of the center of pressure

$$SM = CG - CP$$

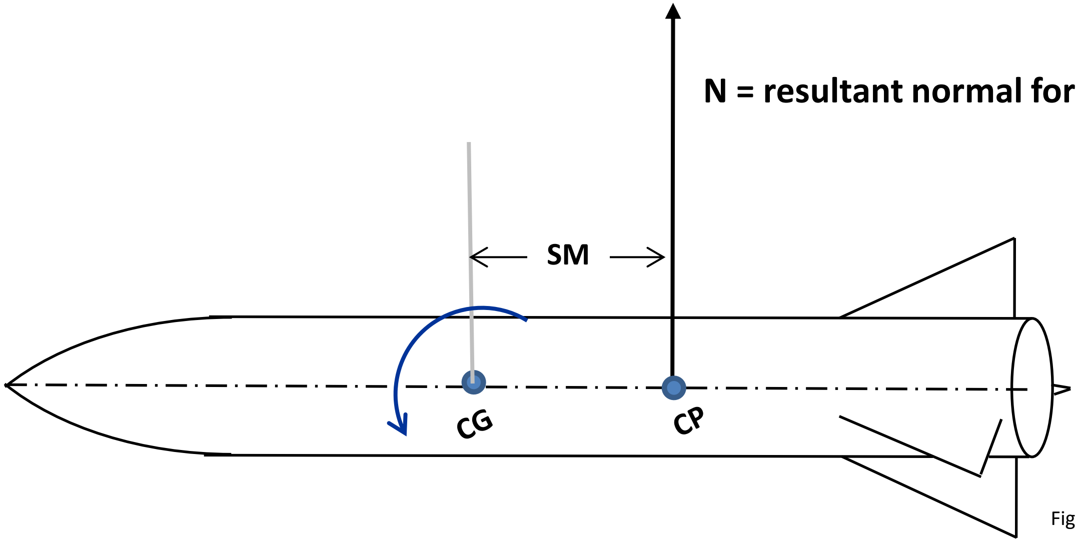
  - The interceptor is unstable when $SM > 0$
  - CG must be forward of CP
  - Rule of thumb: $SM \cong -0.50\,d$ (50% of the missile diameter)
- ☐ Center of Pressure is the centroid of the longitudinal normal force distribution
  - Shifts due as a function of Mach and angle of attack
  - Shift can be minimized using long slender wings
- ☐ Center of Gravity is the centroid of the gravitational forces acting on a body
  - Shifts as a function of propellant burn off

Static Margin

- In a stable configuration ($SM < 0$), the static margin is directly related to a resultant moment about CG

$$m = N\,(SM)$$

- Controlling that resultant moment is critical to ensure stability is maintained
  - It does not have to be zero, just controlled



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Trim Condition

- ☐ Refers to the condition at which the aerodynamic moments on the interceptor are balanced by the control surfaces
- ☐ The amount of tail deflection (δ) required to trim an interceptor is a function of SM and the tail control effectiveness, C_N/δ
- ☐ Tail deflection required for trim

$$\delta = \frac{(C_N/\alpha)\ \alpha\ SM}{(C_N/\delta)\ L_T}$$

  - Where:

    L_T is the distance from the CG to the tail control surface

- ☐ Since maximum tail deflection is fixed, a large static margin reduces the maximum trim angle of attack that can be achieved
- ☐ Large static margin is very stable, but provides a sluggish interceptor response

Deriving the Trim Condition

- ☐ Define the moments due to forces N and ΔN_t

$$m = N\ (SM) = \left(\frac{C_N}{\alpha}\right) \alpha\ Q\ S_{Ref}\ (SM)$$

$$\Delta \mathrm{m_t} = \Delta \mathrm{N_t}\ \mathrm{L_T}$$

$$\Delta m_t = \left(\frac{C_N}{\delta}\right) (-\delta)\ Q\ S_{Ref}\ L_T$$

- ☐ Total moment about CG,

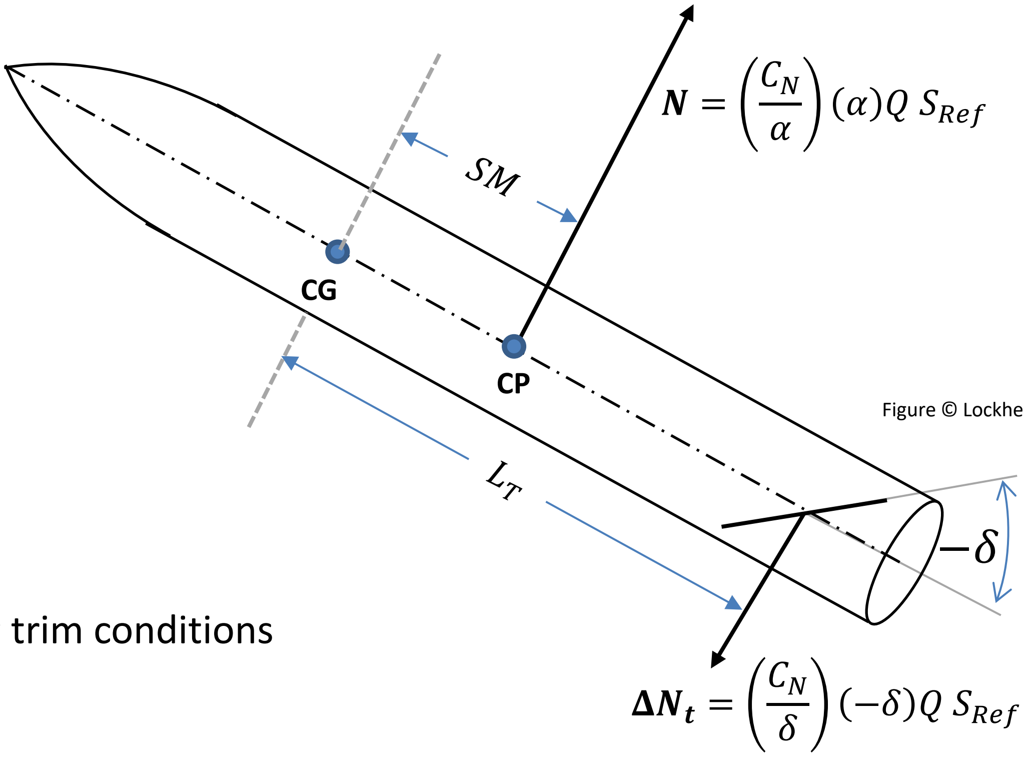
$$m_{CG} = m + \Delta m_t$$

- ☐ For trim, $m_{CG} = 0$,

$$0 = m + \Delta m_t$$

- ☐ The fin deflection required for trim conditions

$$\delta = \frac{(C_N/\alpha)\ \alpha\ (SM)}{(C_N/\delta)\ L_T}$$



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Missile Body Design Trade Space

- The missile designer must
  - Consider the aerodynamic configuration and estimate basic aerodynamic parameters needed for performance and stability analysis
  - Determine the propulsion system requirements considering the mission requirements for range, speed, time-to-intercept and maneuverability
- The final missile design is a compromise of the requirements and preferred approaches of the following design disciplines
  - Aerodynamics
  - Propulsion
  - Structural design
  - Stability, Guidance and Control
  - Thermodynamics
  - Trajectory kinematics
  - Lethality (warhead – fuze)

Missile Design Variation

- As a result, there is tremendous variation in missile design

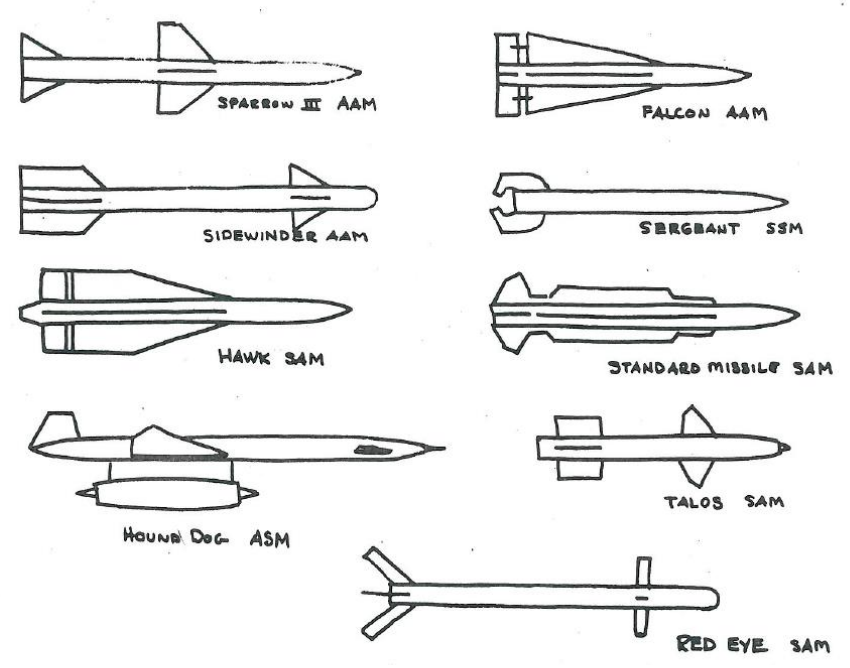
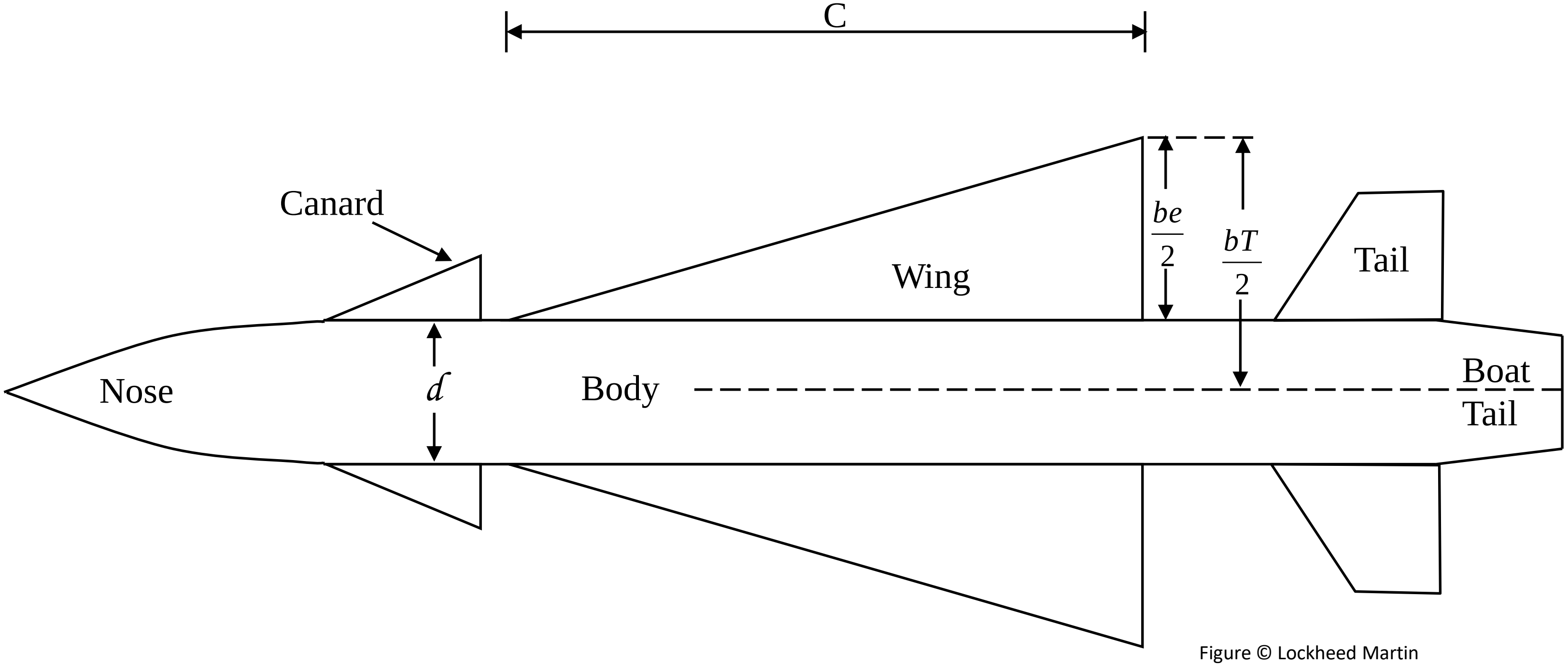


Illustration taken from reference 1

The Missile Design Engineer's Optimum Design is Really the "Least Worst" Design

External Missile Body

- ❑ Illustration of the major missile body parts that affect missile aerodynamics



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Body

- ☐ One of the most important design decisions is body diameter
- ☐ There are very few drivers for having a small missile body diameter
  - ➢ Decreases drag
  - ➢ Launch platform capability
- ☐ There are many drivers for having a large missile body diameter
  - ➢ Increases seeker performance
    - ▪ Higher resolution
    - ▪ Lower noise
  - ➢ Increases blast fragmentation and warhead effectiveness
    - ▪ Larger diameter ⇨ higher velocity fragments
  - ➢ Subsystem packaging (more room for your stuff)
  - ➢ Reduced CG travel and greater bending stiffness

The Desire for a Low Drag Airframe Results in a Small Missile Body Diameter Despite All the Reasons for a Large Body Diameter

Body Fineness Ratio

Body

- Body fineness ratio (BFR) is the ratio of the missile length to the missile diameter

$$BFR = \frac{length}{diameter}$$

- Typical range of the BFR is 5 to 25
  - Portable anti-armor missile have low BFR (Javelin)
  - Air-to-air missiles have high BFR (AMRAAM)
- Benefits of a large body diameter
  - Improved seeker and warhead effectiveness
  - Reduced CG movement during rocket motor burn
  - Greater bending stiffness
  - More internal volume
  - Reduced pitching and yawing inertia
- Benefits of a small body diameter
  - Missile drag

Nose

- ☐ Nose shape requirements are driven by the presence of a seeker
- ☐ Missiles without a seeker can use a pointed nose cone
- ☐ Missiles with a seeker require a trade-off between low drag and good radome characteristics for accurate sensor measurements

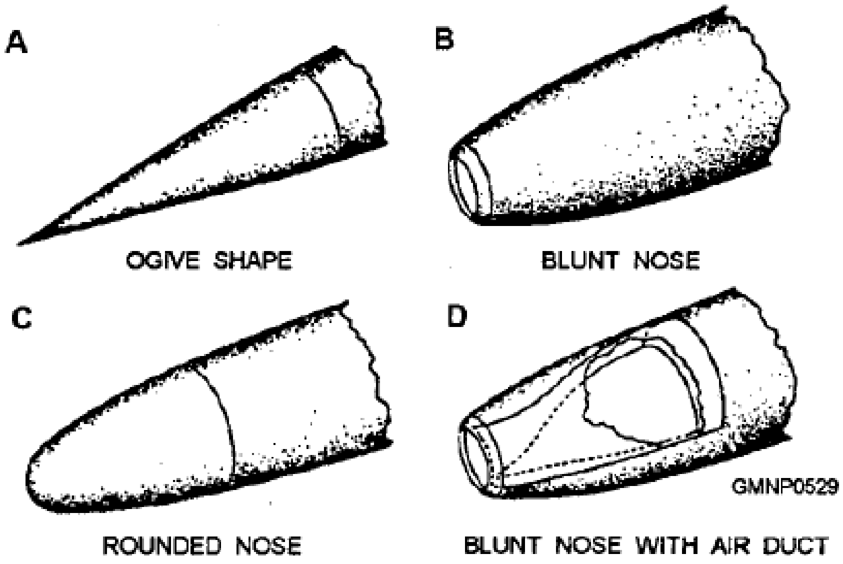


Illustration from reference 2

Nose Fineness Ratio

Nose

- ☐ Nose fineness ratio (NFR) is the ratio of the nose length to the max. nose diameter

$$NFR = \frac{length}{diameter}$$

- ☐ Benefits of a high NFR (~5)
  - Excellent aerodynamics
  - Low observables
  - Reduces wave drag
- ☐ Benefits of a low NFR (~0.5)
  - Minimal radome slope errors
- ☐ Typical range of the NFR is 2 to 4 for supersonic missiles

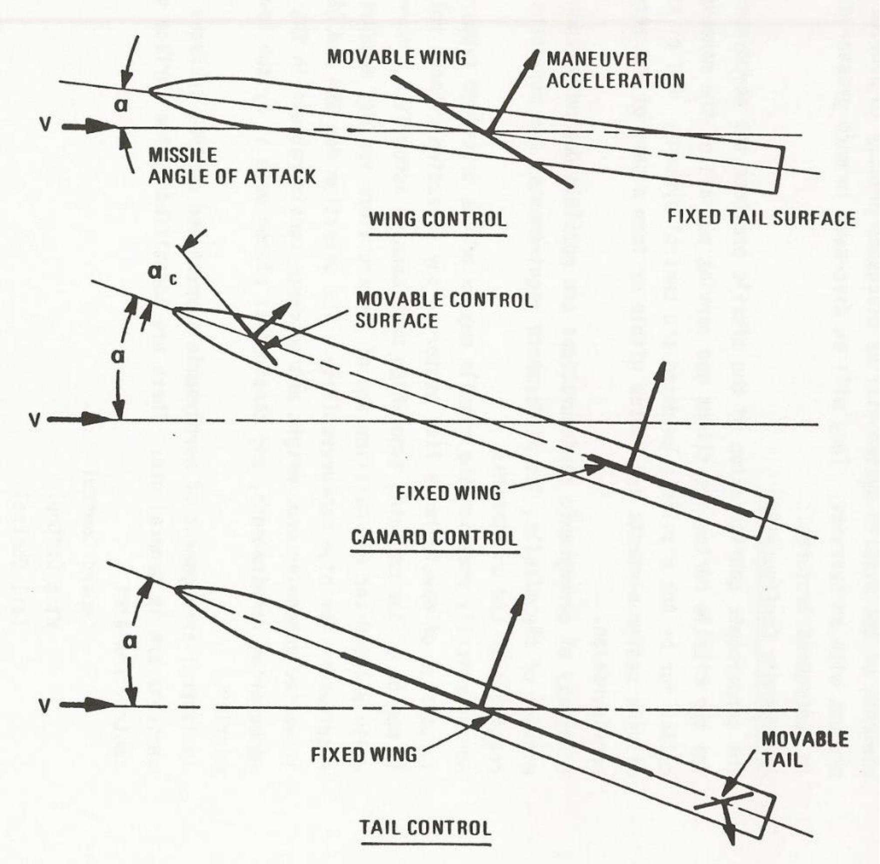
Airfoils

- Airfoils are the wings, fins, etc. that are attached to the missile body
  - Provide flight stability
  - Provide lift
  - Control the missile’s flight path (movable surface)
- The traditional missile body has 3 potential control surfaces
  - Canard
  - Wing
  - Tail
- Taking advantage of the unique control surfaces of a missile allows one to
  - Create a robust guidance law
  - Design for optimal conditions
  - Improve performance

Control Surfaces

Airfoils

View from Side



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View from Aft

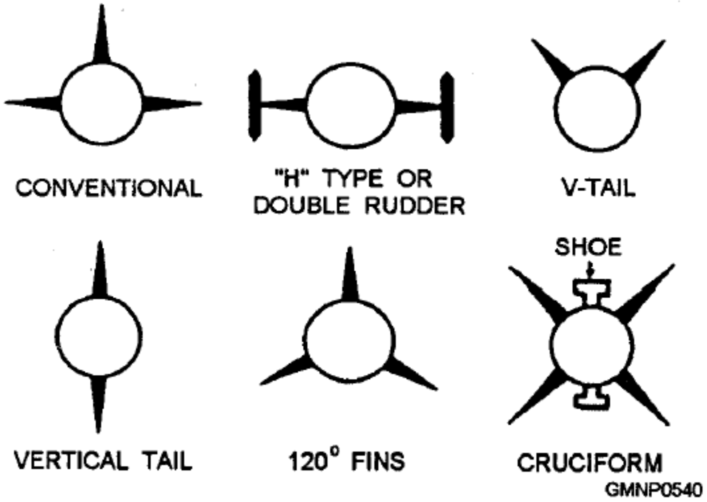


Illustration from reference 2

Control Surface Characteristics

Control Type	Characteristics	Pro	Con
Wing	• Low angle of attack during the maneuver	• Desirable for air breathing engines (because high α may cause the engine to stall)	• Significant structural weight • Large actuation system needed to move the wing
Canard	• Lower α than tail, higher than wing	• Limited to max $\alpha = 45°$ • Physical location is where guidance equipment is normally located	• Limited a capability due to control surface saturation • $\alpha_{canard} = \delta_{canard} + \alpha_{body}$
Tail	• Vector force produced by control surface is opposite of the desired maneuver	• High α capability • $\alpha_{tail} = \delta_{tail} - \alpha_{body}$ • Good physical location (no competition)	• Lift produced by wing and body must overcome tail force to achieve desired acceleration

Control Surface Requirements

- Lift effectiveness (C_N/α) is a must
  - High wing/lift with a low angle of attack
  - High missile response to commands
  - Low drag penalty
- Well thought out control surface positioning
  - Designed for good control and stability
  - Must provide minimum torque due to airloads (location of hinge line)
- Minimum interference of wings with armament effectiveness
  - Wings are often set aft of the warhead

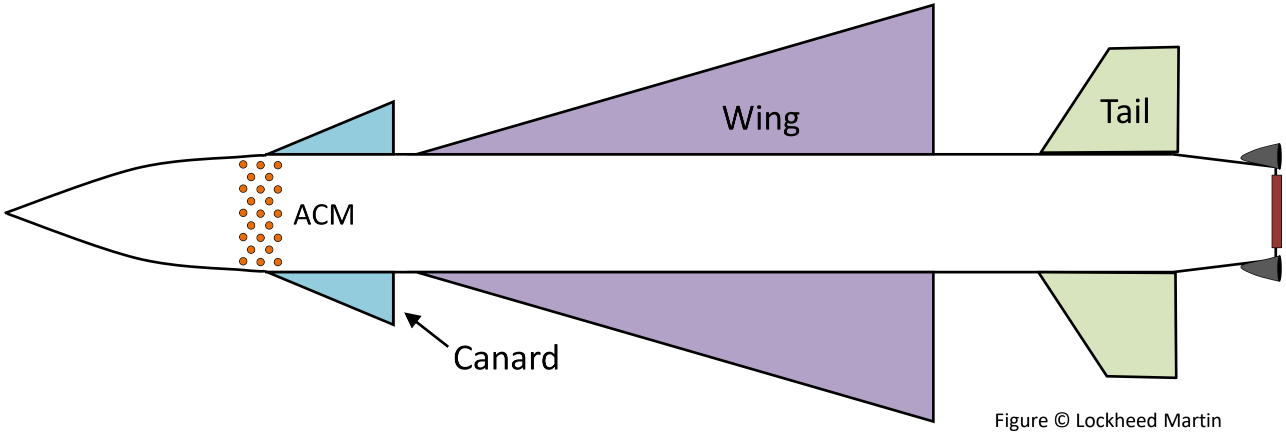
Additional Control Techniques

- Non-traditional means of controlling the missile flight are also common
- Thrust controllers
  - Thrust Vector Actuator (TVA)
    - Nozzles typically located at the rear of the missile which generate a directional thrust for the purpose of changing missile flight path
    - May be the main rocket motor or an auxiliary thrust for the flight path control only
  - Thrust Vector Controller (TVC)
    - Vane inserted into (or just outside) of the rocket motor nozzle to deflect thrust
- Attitude control motors
  - Small explosives in the missile which are fired to generate a nearly instant change angle of attack, thus inducing an normal force

Missile Control Techniques

Illustrated

- ☐ A missile’s flight is controlled through the deflection of various missile surfaces
  - Tail
  - Wing
  - Canard
- ☐ Additional means of controlling flight are also possible
  - Thrust Vector Actuator – Jets located near the tail of the missile (TVA◄)
  - Thrust Vector Control – Surface which deflects the thrust of the rocket motor (TVC ▮)
  - Attitude control motors (ACM ●)



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Wing Size

- Benefits of small or no wings
  - Increased range in high supersonic flight / high dynamic pressure
  - Provides better stability and control at high angles of attack
  - Lower radar cross section (RCS)
  - Launch platform compatibility (packaging, storage)
- Benefits of larger wings
  - Increased range in subsonic flight / low dynamic pressure
  - Lower guidance time constant (more maneuverable)
  - More capability at higher altitudes
  - Less seeker error due to dome error slope (lower angle of attack)
  - Lower gimbal requirements for seeker

References

1. Missile System Engineering Fundamentals. *Missile Aerodynamics*., Lockheed Martin Course, ~1984
2. NAVEDTRA 14110, *Gunner's Mate 1 & C*. November 1996
3. Fleeman, Eugene. *Missile Design Guide*. American Institute of Aeronautics and Astronautics, Inc. 2022