Homework 5 - Aidan Sharpe

Problem 1

In a binary communication system, let the receiver test statistic of the received signal be r_0. The received signal consists of a polar digital signal plus noise. The polar signal has values $s_{01} = A$ and $s_{02} = -A$. Assume that the noise has a Laplacian distribution:

$$f(n_0) = \frac{1}{\sqrt{2}\sigma_0} e^{-\sqrt{2}|n_0|/\sigma_0}$$

where σ_0 is the RMS value of the noise, $f(n_0)$ is the probability density function (PDF), and n_0 is the signal. In the case of a PDF of s_{01} and s_{02}, replace n_0 by $r_0 - A$ and $r_0 + A$. The shape of the PDF for s_{01} and s_{02} is the same. Find the probability of error P_e as a function of A/σ_0 for the case of equally likely signaling and V_T having the optimum value.

Given the two PDFs corresponding to s_1 and s_2, the probability of a bit error is the same as the area of the intersection of the two PDFs, as seen in figure 1.

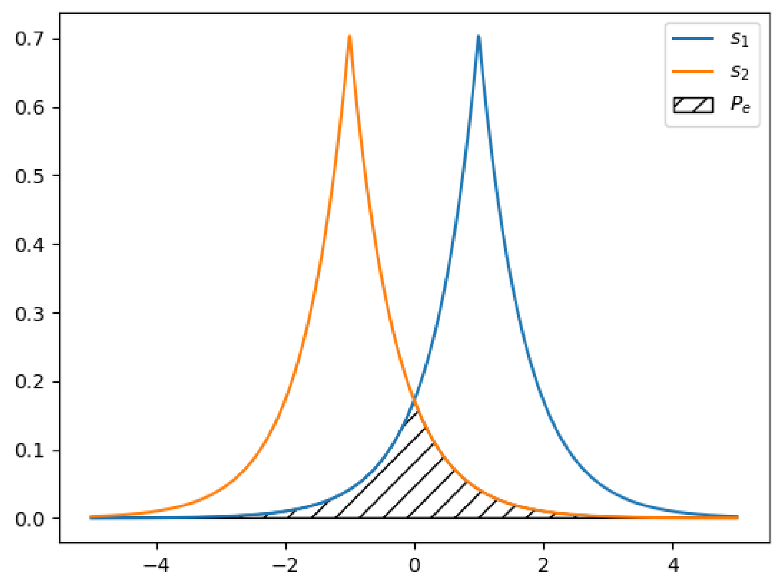

Figure 1: PDFs corresponding to bit error probability

The curve traced out by this area is

$$p(r_0) = \frac{1}{2}\left[f(r_0|s_1) + f(r_0|s_2) - \|f(r_0|s_1) - f(r_0|s_2)\|\right]$$

to find the area under the curve (the probability of a bit error P_e), we integrate $p(r_0)$ for all values of r_0. Since $f(r_0|s_1)$ and $f(r_0|s_2)$ are PDFs, the area under each must be unity. Therefore, distributing the $\frac{1}{2}$ term, we are left with:

$$P_e = 1 - \frac{1}{2}\int_{-\infty}^{\infty} |f(r_0|s_1) - f(r_0|s_2)|\, dr_0$$

Finally, we expand and simplify the integral to:

$$P_e = 1 - \frac{\sqrt{2}}{4\sigma_0}\int_{-\infty}^{\infty} \left|e^{-\sqrt{2}|r_0-A|/\sigma_0} - e^{-\sqrt{2}|r_0+A|/\sigma_0}\right| dr_0$$

After evaluating the intergral, we find that $P_e = e^{-\sqrt{2}A/\sigma_0}$.

Problem 2

A digital signal with white Gaussian noise is received by a receiver with a matched filter. The signal is a unipolar non-return to zero signal with $s_{01} = 1$[V] and $s_{02} = 0$[V]. The bit rate is 1 Mbps. The power spectral density of the noise is $N_0/2 = 10^{-8}$[W/Hz]. What is the probability of error P_e? Assume the white Gaussian noise is thermal noise.

For a unipolar signal received by a receiver with a matched filter, the probability of error is given by:

$$P_e = Q\left(\sqrt{\frac{A^2T}{4N_0}}\right)$$

where $A = 1-0 = 1$ is the amplitude and $T = 1[\mu\text{s}]$. Therefore, $P_e = 2.03 \times 10^{-4}$.