ECOMMS Homework 2 - Aidan Sharpe

Problem 1

```python
import numpy as np

f_c = 1250
f_m = 125
A_c = 10
a = 1

def g(t):
    return A_c * (1 + a*(0.2*np.cos(2*np.pi*f_m*t) + 0.5*np.sin(2*np.pi*f_c*t)))

# First derivative of g(t)
def dg_dt(t):
    return -0.2*2*f_m*np.pi*A_c*a*np.sin(2*np.pi*f_m*t) \
           + 2*np.pi*0.5*f_c*A_c*np.cos(2*np.pi*f_c*t)

# Second derivative of g(t)
def ddg_dtt(t):
    return -0.2*(2*np.pi*f_m)**2*A_c*a*np.cos(2*np.pi*f_m*t) \
           - 0.5*(2*np.pi*f_c)**2*A_c*a*np.sin(2*np.pi*f_c*t)

# Use Newton's method to find the maximum of the function
def newton_method(t):
    for i in range(10):
        t = t - dg_dt(t)/ddg_dtt(t)
    print("Newton method:", t, np.max(g(t)))
    return np.max(g(t))

def dc_dt(t):
    return A_c*a*np.pi*f_c*np.cos(2*np.pi*f_c*t)

# Sample g(t) at the maxima and minima of the carrier signal
def sample_method():
    samples = f_c / f_m
    n = np.arange(samples)
    t = (2*n + 1) / (4*f_c)
    t = t[np.argmax(g(t))]
    print("Sampling method:", t, g(t))
    return t

if __name__ == '__main__':
    t_max = sample_method()
    A_max = newton_method(t_max)
```

```
a_coeff = (A_max - A_c) / A_c
print("Value of a where positive modulation is 90%:", 0.9/a_coeff)
```

Sampling Method

$t_max = 0.0002$, $g_max = 16.9754$ ### Newton Method $t_max = 0.000199206$, $g_max = 16.9755$

Positive modulation is 90% when $a = 1.2902$

$$50000 = \frac{1}{2}\left(\frac{A_c^2}{50}\right)$$

$$A_c = \sqrt{2(50)(50000)} = 2236\text{ V}$$

2b

$A_{min} = A_c(1 + A_1\cos(\omega_1 t_{min}) + A_1\cos(2\omega_1 t_{min}))$

$A_{max} = (2A_1 + 1)A_c$

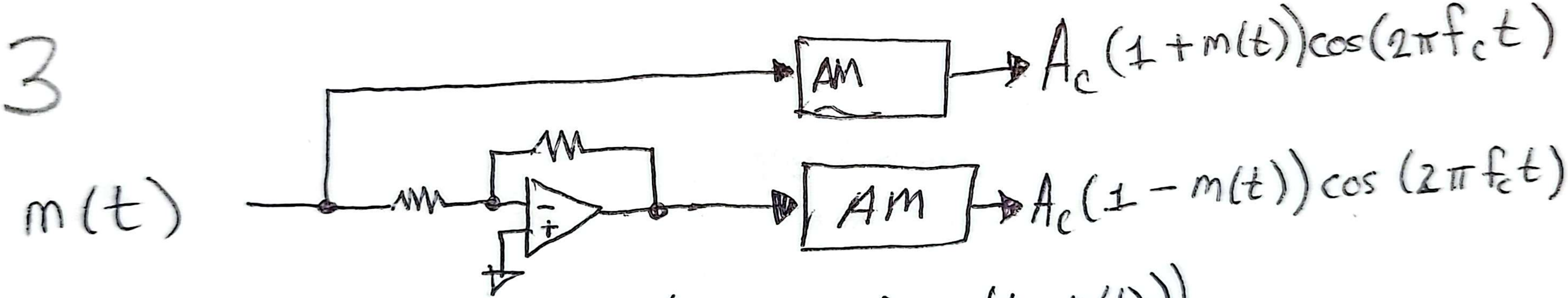
$0.9 = \dfrac{A_{max} - A_{min}}{2A_c} = 2A_1 + 1 - 1 + A_1\cos(\omega_1 t_{min}) + A_1\cos(2\omega_1 t_{min})$

$0.9 = A_1(2 + \cos(\omega_1 t_{min}) + \cos(2\omega_1 t_{min}))$

$t_{min} = 0.0005804\,3$

$A_1 = 1.02857$

3



$$A_c \cos(2\pi f_c t)\left((1+m(t)) - (1-m(t))\right)$$

$$= 2A_c m(t)\cos(2\pi f_c t)$$

#4 $G(f) = \mathcal{F}\{g(t)\}$

$$= \begin{cases} 2M(f) & f<0 \\ M(f) & f=0 \\ 0 & f>0 \end{cases}$$

$$= 2u(-f)M(f)$$
$$= M(f) + M(f)[2u(-f)-1]$$

$$g(t) = F^{-1}\{M(f) + M(f)[2u(-f)-1]\}$$

$$= m(t) + m(t) * F^{-1}\{2u(-f)-1\}$$
$$= m(t) - m(t) * F^{-1}\{2u(f)-1\}$$
$$= m(t) - j\left[\frac{1}{\pi t} * m(t)\right]$$

$$= m(t) - j\hat{m}(t)$$

5

a) $m(t) = 5\cos(\omega_1 t)$
$\hat{m}(t) = 5\sin(\omega_1 t)$

b) $g_L(t) = 5\cos(\omega_1 t) - j5\sin(\omega_1 t)$

c) $g_{RMS}^2 = \frac{1}{T}\int_{-T/2}^{T/2} g_L(t)^2\,dt = 5/\sqrt{2}$

d) $\max(g_L(t)) = 5$

e) $\langle s^2(t)\rangle = \frac{1}{2}\langle |g(t)|^2\rangle = \frac{25}{2}$

f) $P_{PEP} = \frac{1}{2}(\max(g(t)))^2 = \frac{25}{2}$

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